**MA573 - Fall 2018 Midterm Guide and suggested practice exercises**

In short: Strogatz CH.1 - 5, Perko Ch. 1, and first few sections of Strogatz Ch6.

Below is a rough list of topics we've covered in class so far. In general on the midterm, you may be ask to define/explain a few concepts or definitions, and solve problems similar to those done in class and in the homework sets.

## Chapter 2: 1-D flows

- Geometric viewpoint of differential equations: Vector field, phase space, trajectories, equilibria, asymptotics
- Fixed points, stability, linear stability analysis
- Existence and Uniqueness: General conditions, examples of blow-up and non-uniqueness
- Impossibility of Oscillations in 1-D
- Numerical methods

## Chapter 3: Bifurcations

- Basic bifurcations: Saddle-Node, Transcritical, Pitchfork (Super/subcritical), general conditions on vector field for each of these
- Use Taylor expansions to describe bifurcation diagram near change in stability of equilibrium
- Stability diagrams, imperfect bifurcations, cusp points
- How to conclude qualitative dynamics of equations as parameters change: General process: Find equilibria, study their stability using linear and graphical analysis, study change with parameters, conclude asymptotics of initial data in different regions of phase space.
- Non-dimensionalization and Scaling,

## Chapter 4: Flows on the circle

- Vector fields the circle: basic examples, how dynamics can be different than on real line,
- Non-uniform oscillatory: $u' = w + asin(u)$, dynamics and saddle-node dynamics
- Passage time near a ghost of a fold

## Linear Systems: Strogatz Chapter 5 (2-D), Perko CH1 (n-D)

- 2D dynamics: how to classify any linear system using eigenvalues and eigenvectors, draw phase portrait. How to calculate matrix exponential, use eigenbasis (and generalized eigenbasis), to change matrix to matrices of form [a 0; 0 b], [a 1; 0 a], [a -b; b a], and calculate the exponential of those

- Matix exponential in general, series definition, how to calculate, how change-of-basis matrices interact with it,
- Diagonalization: in case where n x n matrix A has n linearly independent eigenvectors
- How deal with non-real complex-conjugate pairs of eigenvalues.
- Definitions of algebraic multiplicity, geometric multiplicity, generalized eigenvectors
- Semi-simple + Nilpotent definition, how to use it to calculate matrix exponential in simple examples, n = 2, 3, 4

### Chapter 6: Phase-plane analysis (May not get to all of these)

- 6.1 General concepts in 2-D phase portraits: Trajectories, closed orbits, fixed points, nullclines, and drawing vector fields,
- 6.2 Existence, uniqueness, and topological consequences: -¿ Fact that trajectories don't cross, closed orbits separate $\mathbb{R}^2$ into invariant regions.
- 6.3 Fixed points and linearization: Don't need to worry about small nonlinear terms yet.

### Some suggested exercises

There will be no homework due on 10/24. Instead, here are a few practice problems on Stable/unstable/center subspaces, and basic phase-plane analysis. For the other topics above, see problems in the corresponding sections of Strogatz and Perko.

(i) Solve the system

$$\dot{x} = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 6 \end{pmatrix} x,$$

and find the stable unstable and center subspaces, $E^s, E^u, E^c$. Use these to sketch the phase-portrait. What are the asymmptotics for the initial condition $x_0 = (0, 0, .1)^T$ as $t \to \pm\infty$? What about $x_0 = (-2, -4, 1)^T$?

(ii) (Strogatz 6.1.1 - 3): Find the fixed points, sketch the nullclines, and the vector-field for the following vector fields. Use linear stability analysis to study the stability of each equilibria, sketch the associated linear vector field, and compare with the nonlinear vector field near each equilibria.

1. $\dot{x} = x - y$, $\dot{y} = 1 - e^x$
2. $\dot{x} = x - x^3$, $\dot{y} = -y$
3. $\dot{x} = x(x - y)$, $\dot{y} = y(2x - y)$