MA573 - Fall 2019 Homework 10 - Due November 15th

Hirsh-Smale-Devaney Problems

Chapter 8: 12 (note: here, unless you really want to, you do not have to write down the specific equations defining the system, a drawing will suffice)

Chapter 9: 1(a,b,d), 2, 11,

Non-textbook Problem(s): Problem 1: Say an equation $X' = F(X), \quad X \in \mathbb{R}^2$ has a conserved quantity $H : \mathbb{R}^2 \to \mathbb{R}$ which is non-constant on any open set in $\mathbb{R}^2$. Show that this system has no asymptotically stable equilibria.

Problem 2: Consider the damped oscillator $x'' + bx' - x + x^3 = 0$, write this equation as a first-order system. For $b = 0$, find a conserved quantity. Now, look study the system for $0 < b \ll 1$, sketch the basin of attraction for the equilibrium $(1, 0)$.

Problem 3: Consider the system

$$\begin{aligned} x' &= y \\ y' &= x - x^2 \end{aligned}$$

- Perform the standard linearized analysis around any equilibria, and characterize the vector field using null-clines
- Construct a conserved quantity for this flow. Is this a Hamiltonian system?
- Using the level sets of your conserved quantity sketch the phase portrait. Locate an *Homoclinic* orbits, that is, any trajectories which converge in both forwards and backwards time, $t \to +\infty$, to an equilibrium.