

MA573 - Fall 2019 Homework 8 - Due November 1st

Hirsh-Smale-Devaney Problems

Chapter 8: 1(i,ii,iv,v), 2 (hint: see examples in Sec. 8.3 on pg. 173, as well as in 8.1 pg).

Bonus problem: 11 (4 pts of extra credit)

Non-textbook Problem(s):

Problem 1: (*Projective coordinates*) Consider the linear system

$$\begin{aligned}x' &= x - x^3, \\y' &= -y - x^2.\end{aligned}\tag{0.1}$$

- (i) Find all equilibria and classify each associated linear system. If possible, use the Linearization Theorem (pg. 168 of text) to also classify and make a rough sketch the nonlinear phase portrait in a local neighborhood of each equilibrium.
- (ii) The origin should be a saddle. Describe the stable and unstable subspaces of it's linearization.
- (iii) Using the method we discussed in class, find the 4th-order Taylor expansion of the function $h^u(x)$ which describes the local unstable manifold $W^u(0) = \{(x, y) \mid x = h^u(y)\}$. Use this to more accurately sketch the local phase portrait near the origin.
- (iv) Do the same for the stable manifold $W^s(0)$. Note here the graph function should have the form $x = h^s(y)$.

Problem 2: Now consider the slightly altered system

$$\begin{aligned}x' &= x - x^3, \\y' &= -y - x^3.\end{aligned}\tag{0.2}$$

The linearization about the origin should not change. Does the unstable manifold $W^s(0)$ change?