MA573 - Fall 2019 Homework 9 - Due November 8th

Hirsh-Smale-Devaney Problems

Chapter 8: 5,9

Non-textbook Problem(s):

For the next two problems, consider the again the nonlinear system you studied last homework.

$$\begin{aligned} x' &= x - x^3, \\ y' &= -y - x^2. \end{aligned} \tag{0.1}$$

Problem 1:(dynamics on the stable manifold) Using the taylor series expansion for the stable manifold $W^s(0)$ derive a 1-D differential equation which approximates the flow on the invariant manifold. (Hint: use the fact that $x = h^s(y)$ on this manifold and one of the equations making up the system (0.1). Describe the flow on the manifold.

Problem 2: Investigate the stable manifold numerically using matlab. In particular, using a collection of trajectories, try to approximate the stable manifold in a neighborhood of the origin. Compare this to your expansion found in HW 8. (For example, taking inspiration from the proof of the "stable curve theorem" in the text, consider a line of initial conditions $\{(x, y)\, y = \pm\delta\}$ and flow them forwards in time, locate which initial conditions diverge to the left? to the right? Do this for several δ values to get an approximation of $W^s(0)$).

Problem 3: Next let $f_a(x) = x(x-1)(x-a)$ and consider

$$\begin{aligned} x' &= y \\ y' &= f_a(x) - cy \end{aligned} \tag{0.2}$$

(a) Set $c = 0$, find all equilibria and classify their linearized and nonlinear (if possible) phase portraits for $a \in [0, 1]$.

(b) Now set $a = 1/2$ and study qualitative changes for the linear and nonlinear phase portraits about equilibria for $c \geq 0$.

(c) Possibly after some numerical investigation, still with $a = 1/2$, sketch the qualitatively different phase portraits for $c \geq 0$.