MA573 - Fall 2019 Midterm Guide

In short: Hirsh-Smale-Devaney Chapters 1 - 6,

Below is a rough list (though not necessarily exhaustive) of topics we've covered in class so far. In general on the midterm, you may be ask to define/explain a few concepts or definitions, and solve problems similar to those done in class and in the homework sets.

Chapter 1: First Order Equations

- Basic concepts: differential equation, solution, initial value problem, phase-line, equilibria, autonomous/non-autonomous, linear/nonlinear equations, converting higher-order differential equations into a first order system.
- Simple 1-D bifurcations
- Poincare Map, periodicity, periodic orbits
- Numerical algorithms: Euler, improved Euler, Runge-Kutta 4

Chapter 2: Planar Linear Systems

- Concepts: Vector field, direction field, Phase Plane, trajectories, asymptotics of solutions
- 2-D linear algebra: solving 2-D equations, linear independence, determinants, eigenvalue, eigenvectors
- general solution of linear systems, linearity principle

Chapter 3: Phase portraits for Planar systems

- How to draw phase portraits for different cases: spiral source/sink/center, saddle, source, sink, degenerate (where at least $\lambda = 0$), repeated roots
- Cannonical coordinate changes using eigenbases

Chapter 4: Classification of Planar Linear Systems

- How to use trace-determinant plane to quickly classify 2-D systems
- General concept of topological conjugacy

Chapter 5: Higher dimensional linear algebra

- Linear algebra: Spanning set, basis, subspace, linear independence, kernel, range, determinant, canonical form of matrices (sometimes referred to as Matrix Diagonalization)
- Complex Eigenvalues
- Repeated Eigenvalues, generalized eigenvectors,

Chapter 6: Higher dimensional linear systems

- Solving and analyzing linear systems, plotting phase portraits
- Distinct eigenvalues, repeated eigenvalues, purely-imaginary eigenvalues,
- Using polar coordinates to reduce/simplify systems
- Harmonic Oscillators
- Matrix Exponential

Some suggested exercises

There will be no homework due on 10/18. Instead, here are a few practice problems on Stable/unstable/center subspaces, and basic phase-plane analysis. For the other topics above, see problems in the corresponding sections of Hirsh-Smale-Devaney. Another source of practice problems is the text by Strogatz.

(i) Solve the system

$$\dot{x} = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 6 \end{pmatrix} x,$$

and find the stable unstable and center subspaces, E^s, E^u, E^c. Use these to sketch the phase-portrait. What are the asymmptotics for the initial condition $x_0 = (0, 0, .1)^T$ as $t \to \pm\infty$? What about $x_0 = (-2, -4, 1)^T$?

(ii) Consider the system

$$X' = \begin{pmatrix} a & 0 & 1 \\ -1 & 1 & -1 \\ -1 & 0 & a \end{pmatrix} X,$$

Classify the dynamics for various values of a and find any bifurcation values, how do E^s, E^u, E^c change as a is varied? Sketch the phase-portraits. Do the same for the matrix

$$X' = \begin{pmatrix} a & 0 & 1 \\ -1 & 1 & -1 \\ -1 & 0 & -a \end{pmatrix} X,$$

(iii) Hirsh/Smale/Devaney: 6.4, 6.12, 6.13, 6.15,