MA771 - Fall 2020 Homework 2 - Due Friday, October 9th

Please scan or type your homework and email it to me by the due date. Homeworks will be graded electronically and emailed back to you.

Barreira and Valls Problems

Chapter 3: 16

Chapter 4: 7, 10,

Additional Problems :

Problem 1: Find a conjugacy between the map $q(x) = 2x^2 - 1$ on $[-1, 1]$ and the Tent map

$$T(x) = \begin{cases} 2x, & x \in [0, 1/2] \\ 2 - 2x, & x \in [1/2, 1]. \end{cases}$$

defined on $[0, 1]$. (Hint: Consider trigonometric functions)

Problem 2: Calculate the entropy of the mapping $f(x) = x(1 - x)$ on $[0, 1]$.

Problem 3: Consider the standard map $f_{\omega,\epsilon}(x) = x + \omega + \frac{\epsilon}{2\pi}\sin(2\pi x)$ posed on the circle. Recall we showed that this is a homeomorphism for $|\epsilon| < 1$.

- For fixed $\epsilon \neq 0$, show that $\rho(f_{\omega,\epsilon})$ is non-decreasing in ω.
- Suppose that $\rho(f_{\omega_0,\epsilon}) = p/q,\ p, q \in \mathbb{Z}$ for some ω_0. Show there is an interval I_0 containing ω_0 such that $\rho(f_{\omega,\epsilon}) = p/q$ for all $\omega \in I$.
- Now fix ϵ, not too small, and numerically measure $\rho(f_{\omega,\epsilon})$ for $\omega \in [0, 1]$. Plot your result. It might help to try a few fixed values of ϵ here and you may use a numerical software of your choice (please speak to me if you'd like some guidance getting started on the numerics).
- (Bifurcation): For $\epsilon \in (0, 1)$ fixed, describe how the set of fixed points of $f_{\omega,\epsilon}$ behaves as the parameters ω, ϵ are varied.
- (optional) Using graphical analysis, numerics, or another method, investigate how the set of periodic points varies as both ω and ϵ are varied.