

MA771 - Fall 2020 Homework 3 - Due Friday, October 23rd

Please scan or type your homework and email it to me by the due date. Homeworks will be graded electronically and emailed back to you.

Problem 1: Choose one of the following problems:

(a): [Devaney Exc. 1.18.1, Pg. 146] List all possible itineraries for $f_\mu(x) = \mu x(1 - x)$ in the cases where $\mu = 2$ and $\mu = 3$.

(b): Show that the rotation number is a continuous function as a mapping on the space of circle homeomorphisms (with the standard C^0 topology).

Problem 2: Compute the kneading invariant of an infinitely renormalizable unimodal map. (See [Devaney, Exc. 1.18.3 Pg. 146])

Problem 3: Find the parameter value μ_* for the first period-doubling bifurcation in the logistic map $f_\mu(x) = \mu x(1 - x)$, and find an expansion for the period-2 orbit curves $x_1(\mu), x_2(\mu)$ for μ near μ_* .

Problem 4: Using a numerical software of your choice investigate the *orbit diagram* of the logistic map $f_\mu(x) = \mu x(1 - x)$ for $\mu \in [0, 4]$ as follows. Consider 1000 values of μ equidistantly spaced in $[0, 4]$. For each μ value, compute the first 500 iterates of the orbit $\gamma_+(1/2)$. Plot the last 400 iterates $x_j = f_\mu^j(1/2)$ as points (μ, x_j) in the (μ, x) -plane. This gives a view of the chaotic regions and the μ values for which f_μ has an attracting periodic orbit. (Optional): There are "windows" in the chaotic regions where an attracting k -periodic orbit arises, can you say anything about these windows, the period k , and their ordering with respect to μ ?