MA775 - Fall 2017 Homework 1 - Due September 19th

Note, Problems 2 and 5 were from my own graduate ODE course at U. of Minnesota, given by Prof. A. Scheel.

Problem 1. *(Chicone Exc. 1.10)* For each integer p, construct the flow of the differential equation $\dot{x} = x^p$.

Problem 2 *Let Φ be a flow on $X = \mathbb{R}$. Show that $\Phi(t, x)$ is monotone in t for all $x \in X$. Is it monotone in x for all t?*

Problem 3.*(Chicone 1.11) Construct the family of solutions $t \mapsto \phi(t, \xi)$ for the differential equation $\dot{x} = t$ such that $\phi(0, \xi) = \xi$ for $\xi \in \mathbb{R}$. Does ϕ define a flow? In particular, does it satisfy the group property $\phi_t \circ \phi_s = \phi_{t+s}$?*

Problem 4.*(From class) Let $f \in C^k(\mathbb{R}^n, \mathbb{R}^m)$ with k a positive integer and J an interval in $\mathbb{R}$. Define,*

$$\begin{aligned} F : C^0(J, \mathbb{R}^n) &\to C^0(J, \mathbb{R}^m) \\ u(\cdot) &\mapsto f(u(\cdot)), \end{aligned} \tag{0.1}$$

Prove that $F \in C^k$. Optional exercise: Extend this proof for $k = \infty$ and $k = \omega$ (analytic).

Problem 5. *Let E is a complete metric space and P is an open set in $\mathbb{R}^m$. Given a continuous mapping $T : E \times P \to E$ for which there exists a $K < 1$ so that $d(T(u, p), T(v, p)) \leq Kd(u, v)$ for all $p \in P$ and $u, v \in E$, show that there exists a family of fixed points $u_*(p)$ which are continuous in the parameter p. (Note this is the idea of the proof for continuous dependence on parameters for ODEs with vector-field only continuous in p.)*