MA775-Ordinary Differential Equations

Ryan Goh

September 2, 2017

Course Details Time: Tuesday,Thursday, 11:00am-12:15am; PSY B50

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Office hours: TBD

Course Description: Graduate level introduction to ordinary differential equations and dynamical systems. This course aims to give a broad view of the subject by emphasizing key concepts and techniques at the heart of the theory as well as to highlight the many applications of the theory. It is the goal of this course that participants obtain a solid foundational knowledge of the field in order to continue research into the area and/or to apply it to their particular field of study.

We hope to cover

- Basic functional analysis
- Existence and uniqueness of ODE's
- Qualitative concepts in ODE's
- Review of Linear theory and phase plane analysis
- Stability theory and invariant manifolds
- Non-autonomous equations and periodic dynamics
- Global methods in dynamical systems
- Bifurcations and Normal forms

Depending on class preference and time, we may also delve into some of the following topics. If there is another related topic that you require in your own field of study or want to learn more about feel free to contact me.

- Heteroclinic and homoclinic bifurcation theory
- Symmetries and equivariant bifurcation theory
- Infinite dimensional dynamical systems and evolution equations
- Forced oscillations and averaging theory

- Hamiltonian systems
- Singular Perturbations
- Numerical methods

Homework Homeworks will be assigned every 1-2 weeks and will be due in class one week afterwards. You are welcome (and encouraged!) to work together but please make sure to write up solutions on your own.

Project One the most important skills developed in graduate school is the ability to read, comprehend, and synthesize scientific literature. Since the math department of Boston University has a rich history of research in differential equations, dynamical systems, and their applications, your assignment will be to read and work through a research paper of a current or former BU mathematics faculty member which studies or uses techniques from dynamical system theory. Our faculty has diverse interests ranging from fluid dynamics, pattern formation, mathematical neuroscience, to celestial mechanics, so hopefully there is a topic which is at least tangential to your field of study.

You must then write a report which summarizes and explains the paper in your own words. Near the end of the semester, each student will give a presentation to the class on the paper, explaining the paper and putting it in context with what we've discussed throughout the semester. Details

Below is a list of some of the BU faculty and their research interest which may be good candidates. Some suggested papers are posted on the course website with links. If none of the research areas listed below are of interest to you, you may also read a paper of your choosing which is related to ODEs and dynamical systems. If you have no idea what you want to read, feel free to come talk with me to figure out a good paper/area to look into.

A brief overview of some of the BU faculty and their main research interests

- Nancy Copell: Pattern formation, mathematical neuroscience,...
- Margaret Beck: Pattern formation, fluid dynamics, stability theory, coherent structures
- Paul Blanchard: Complex Dynamics, fractals...
- Richard Devaney: Complex Dynamics, Hamiltonian systems, bifurcation theory, general dynamical systems theory
- David Fried: General Dynamical systems theory, mathematical neuroscience, topological dynamics
- Glenn Hall: Celestial mechanics, Hamiltonian dynamics, general dynamical systems theory
- Tasso Kaper: Pattern formation, Singular perturbation theory, Multi-scale dynamics.
- Mark Kramer: Mathematical neuroscience,
- Gene Wayne: Fluid dynamics, partial differential equations, pattern formation, Hamiltonian systems/KAM theory

Note there are many other professors at BU who's research relates to ODEs and dynamical systems theory. Please feel free to take a look at the Faculty page at the math department website to look for other possible options. **If you are having trouble selecting a paper please come talk to me and we can find something of interest to you.**

Requirements

- (By October 3rd): Confirm and approve with me the paper you wish to read.
- Meet with me at least once during the semester to discuss the paper. Of course you are more than welcome to meet with me more to ask any questions or discuss any facet of the paper you choose.
- (November 30th) Submit a draft of your report, after which I will make some suggestions to improve the exposition.
- (December 12th) Turn in a final draft of the paper .
- (Time TBD) Give a presentation in class/student dynamics seminar explaining the paper.

Grades Your course grade will be based on your homework scores (70%) and your project (30%).

References The lectures will be mostly self-contained and you are not required to purchase any of these books. They are helpful in either looking up details or alternate proofs of results we will cover. Any books marked with a (*) have been put on reserve at the BU Science & Engineering Library. More references and links to specific papers relevant to lectures will be posted on the course webpage.

Main References:

- (*) C. Chicone. *Ordinary Differential Equations with Applications.* Texts in Applied Mathematics, 34, Springer 2006. - This book has many alternate proofs of results we'll cover in class.
- (*) J. Guckenheimer and P. Holmes, *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields.* Applied Mathematical Sciences, 42, Springer-Verlag, 1990. - This classic book covers many important concepts, phenomenon, and techniques in the area, mostly without proof.

Other useful references:

- (*) L. Perko, Lawrence. *Differential equations and dynamical systems.* Vol. 7. Springer Science & Business Media, 2013. - In between an undergraduate and graduate level text, has a nice review of the theory of finite dimensional homogeneous linear equations.
- Coddington and Levinson. *Theory of ordinary differential equations.*
- Chow, S-N., and Jack K. Hale. Methods of bifurcation theory. Vol. 251. Springer Science & Business Media, 2012.
- (*) Hale, Jack K., Ordinary Differential Equations. Dover Publications, 1997.
- (*) Arnol'd, Vladimir Igorevich. Geometrical methods in the theory of ordinary differential equations. Vol. 250. Springer Science & Business Media, 2012.
- (*) Palis, J. Jr, and Welington De Melo. Geometric theory of dynamical systems: an introduction. Springer Science & Business Media, 2012.
- Hartman, Philip. Ordinary differential equations. SIAM (2002). (available online via. BU library site)
- (*) Hirsch, Morris W., Stephen Smale, and Robert L. Devaney. Differential equations, dynamical systems, and an introduction to chaos. Academic press, 2012. − A good undergraduate level reference with many nice examples.
- Hale, Jack K., and Kocak, H.. Dynamics and bifurcations. Vol. 3. Springer Science & Business Media, 2012.

- (*) Arnold, V. I. Ordinary differential equations (translated by RA Silverman). MIT Press, Cambridge, Massachusetts 2 (1973): 196-213.