MA775 - Fall 2023 Homework 1 - Due Tuesday, September 26th

Problem 1. *(Chicone Exc. 1.10)* For each integer p, construct the flow of the differential equation $\dot{x} = x^p$.

Problem 2 *Let Φ be a flow on $X = \mathbb{R}$. Show that $\Phi(t, x)$ is monotone in t for all $x \in X$. Is it monotone in x for all t?*

Problem 3. *(Chicone 1.11) Construct the family of solutions $t \mapsto \phi(t, \xi)$ for the differential equation $\dot{x} = t$ such that $\phi(0, \xi) = \xi$ for $\xi \in \mathbb{R}$. Does ϕ define a flow? In particular, does it satisfy the group property $\phi_t \circ \phi_s = \phi_{t+s}$?*

Problem 4. (Teschl Prob. 2.6) Are the following functions Lipschitz continuous near 0? If yes, find a Lipschitz constant for some interval containing 0.

(i) $f(x) = 1 - x$

(ii) $f(x) = |x|^{1/2}$.

(iii) $f(x) = x^2 \sin(\frac{1}{x})$.

Problem 5. *Let E be a complete metric space, with metric d, and P an open set in $\mathbb{R}^m$. Given a continuous mapping $T : E \times P \to E$ for which there exists a $K < 1$ so that $d(T(u, p), T(v, p)) \leq Kd(u, v)$ for all $p \in P$ and $u, v \in E$, show that there exists a family of fixed points $u_*(p)$ which are continuous in the parameter p. (Note you can't use the implicit function theorem here).)*