MA776 - Spring 2022 Homework 3 - Due Friday, Mar 4th

Please scan or type your homework and email it to me by the due date. Homeworks will be graded electronically and emailed back to you.

Evans Problems

Chapter 2: 24

Chapter 3: 4,5, 19,

Additional Problems : For the next three problems consider the initial value problem in scalar Burger's equation

$$\begin{aligned} u_t + \left(\frac{u^2}{2}\right)_x &= 0, \qquad (x,t) \in \mathbb{R} \times \mathbb{R}_+ \\ u &= g, \qquad (x,t) \in \mathbb{R} \times \{0\} \end{aligned}$$

Problem 1: (alternative derivation of rarefaction wave) For a constant $a > 0$, consider the initial condition

$$g_a(x) = \begin{cases} 0, & x \leq 0 \\ x/a, & 0 < x < a \\ 1, & x \geq a \end{cases}$$

Solve the initial value problem for $a > 0$, and take the limit $a \to 0^+$ to derive the rarefaction wave discussed in class.

Problem 2: For a given constant $u_0 > 0$, consider the following initial condition

$$g(x) = \begin{cases} 0, & x \leq 0 \\ u_0(x-1), & x > 0 \end{cases}$$

The integral solution has a jump discontinuity along some curve described by $x = s(t)$. Find this curve and sketch it along with other characteristic curves in the (x,t) plane. Also, determine the integral solution on each side of the shock.

Problem 3: Determine the wave-breaking time and spatial locations for the initial condition $g(x) = \sin x$.

Problem 4 Solve the following initial value problem for $u(x,t) \in \mathbb{R}$ using the method of characteristics

$$\begin{aligned} u_t + (u_x)^2 &= t, \qquad (x,t) \in \mathbb{R} \times \mathbb{R}_+ \\ u(x,0) &= x, \qquad x \in \mathbb{R}, t = 0. \end{aligned}$$