Week 3 Exercises

1. Show that the definition of sheaf on a site corresponds with the classical definition in the case when the site is the one coming from a topological space.

2. Prove if $\varphi : Y \to X$ is a Galois covering, then φ is surjective, finite étale. Then show that if $\varphi : Y \to X$ is surjective, finite, étale and the degree is $\# Aut_X(Y)$, then φ is a Galois covering with group $\mathrm{Aut}_X(Y)$.

3. Prove Prop 6.8. Namely, if $A \to B$ is faithfully flat, then

$$0 \to A \to B \to B \otimes_A B$$

is exact, where the latter map is $b \mapsto b \otimes 1 - 1 \otimes b$.

4. Let $Z \in \mathrm{Sch}/X$ and $\mathcal{F}_Z : X_{et} \to \mathbf{Sets}$ the presheaf defined by the functor of points of Z. Show $\mathcal{F}_Z$ is a sheaf for Zariski coverings.

5. (Coherent Sheaves)

   (a) Let $\varphi : U \to X$ be étale. Let $\mathcal{M}$ be a coherent sheaf on X (in the usual sense). Show that $\varphi^*\mathcal{M}$ is a coherent sheaf on U.

   (b) Define $\mathcal{M}^{et}(U) = \Gamma(U, \varphi^*\mathcal{M})$. (Note: the notation here is a bit misleading, as the input on the left should really be φ rather than just U.) Show that the presheaf $\mathcal{M}^{et}$ is actually a sheaf on X_{et}. (Hint: Use the analogous to above exact sequence $0 \to M \to B \otimes_A M \rightrightarrows B \otimes_A B \otimes_A M$ when $A \to B$ is faithfully flat.)

   (c) Let X be a variety over the field k. Use the classical exact sequence

$$\varphi^*\Omega^1_{X/k} \to \Omega^1_{U/k} \to \Omega^1_{U/X} \to 0$$

   to deduce that $(\Omega^1_{X/k})^{et}$ restricted to U_{zar} is $\Omega^1_{U/k}$.