Week 9 Exercises

The notation in the exercises comes from Milne's *Lectures on Etale Cohomology (v2.21)*..

1. Verify the following statements:
   (a) A finite morphism of schemes is proper.
   (b) A k-variety X is complete if and only if the map $X \to \mathrm{Spec}(k)$ is proper.

2. Each of these examples shows the necessity of one hypothesis in the statement of Corollary 17.8 (which one?).
   (a) Let $P = \mathrm{Spec}(\mathbb{Q})$ and let $\mathcal{F}$ be the constant sheaf $\underline{\mathbb{Z}/2\mathbb{Z}}$. Check that $H^1(P; \mathcal{F})$ is not finite.
   (b) Take $X = \mathbb{A}^1$ over k separably closed of characteristic $p \neq 0$ and let $\mathcal{F}$ be the constant sheaf $\underline{\mathbb{Z}/p\mathbb{Z}}$. Check that statement (b) does not hold for this example.

3. In topology, given a locally compact topological space U and a sheaf $\mathcal{F}$ on U, one defines the cohomology groups with compact support by setting

   $$\Gamma_c(U, \mathcal{F}) = \varinjlim_{Z} \Gamma_Z(U, \mathcal{F})$$

   where Z runs over the compact subsets of U, and defining $H^r_c(U, -) = R^r\Gamma_c(U, -)$.

   This suggest using the analogous definition in algebraic geometry, where now U is an algebraic variety, $\mathcal{F}$ is a sheaf on U_{et}, and Z runs over the complete subvarieties Z of U.

   Convince yourself that this is not the right definition by considering the example where U is any nonsingular affine curve over an algebraically closed field k and $\mathcal{F}$ is the constant sheaf $\underline{\mathbb{Z}/n\mathbb{Z}}$ for some n coprime to $\mathrm{char}(k)$.