

Homework due inclass Friday Feb. 17 Solutions:

1. How many different sequences of heads and tails are possible if you flip a coin 10 times?

Answer Since each coin flip can have 2 outcomes (heads or tails), there are $2 \cdot 2 \cdot \dots \cdot 2 = 2^{10} = 1024 \approx 1000$ possible outcomes of 10 coin flips.

2. If each possible sequence is equally likely, what is the probability of the sequence HTHHTTTHHHT?

Answer Assuming the equally likely outcome model, the probability of this one outcome is $1/1024 \approx 1/1000$.

3. If you repeat the experiment of flipping a coin ten times 10,000 times, (so 100,000 flips in all), about how many times do you expect to get the sequence HTHHTTTHHHT?

Answer Since probability represents “long-range frequency”, we expect this particular sequence to occur about once for every 1000 repeats of the experiment of flipping a coin 10 times. So if we repeat the experiment 10,000 times, then we expect to get this particular sequence about 10 times.

4. Suppose you do just one experiment flipping a coin ten times and you get the sequence HHHHHTTTTT. What is the probability that you got the sequence HHHHHTTTTT? (Read this question carefully– what is the probability that you got HHHHHTTTTT once you know that you got HHHHHTTTTT?)

Answer Remember, probability measures our lack of knowledge– here we do not have any lack of knowledge. If we got the sequence HHHHHTTTTT then the probability that **we got** this sequence is 1 (we know we got it!).

5. Suppose a friend flips a coin ten times and gets HTHTHHTTTH. If your friend says to you “Wow, that was an incredible coincidence. Of all the sequences, I got HTHTHHTTTH. The probability of that sequence is tiny, there must have been something wrong with the coin because I got this incredibly unlikely sequence.” What is the problem with your friend’s statement–in a couple of sentences, explain the error in their thinking.

Answer “But you already know the outcome of your coin flips–you had to get some sequence and you happened to get this one. It is no more or less unlikely than any other sequence. If you had predicted it in advance, then I would be impressed!” (The Rosencrantz and Guildenstern are Dead clip has a bit of dialog that is quite accurate–listen at 8 min 40 seconds to 9 min.)