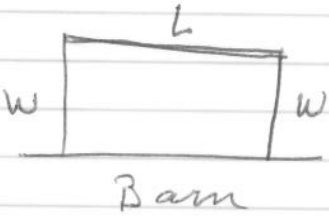
Homework #1 - 1, 2, 3 (Rest done in class).

1. Farmer Jones has 100 feet of fence. She wants to make a rectangular pen using all the fence for three sides and the side of the barn for the fourth side. What is the area as a function of the length.



Solution: Let L = length of pen (in feet) and W = width (in feet).

Then the area A is

$$A = LW.$$

To express this as a function of the length, we note that $2W + L = 100$

(since this is how much fence she has)

So

$$2W = 100 - L,$$

or $$W = 50 - L/2.$$

So the area is (replacing W by $50 - L/2$)

$$A(L) = L \cdot (50 - L/2)$$

or

$$A(L) = 50L - L^2/2$$

} either is OK.

2. A piece of wire 2 meters long is cut into 2 pieces. One piece is made into a square, the other is made into a circle. What is the area enclosed.

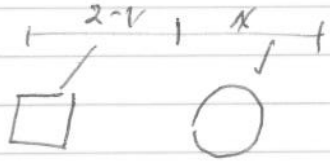
Answer It depends on how you cut the wire -

Let x = length of piece made into circle so

$2-x$ = length of piece made into square.

Side length of square is

$$\frac{2-x}{4}$$



So the area of the square is

$$\left(\frac{2-x}{4}\right)^2.$$

The circumference of the circle is x (the length of wire) since circumference = $2\pi \times$ radius, the radius of the circle is $\frac{x}{2\pi}$ (i.e. $2\pi \times \text{radius} = x$ so $\text{radius} = \frac{x}{2\pi}$).

So the area is $\pi\left(\frac{x}{2\pi}\right)^2 = \pi \frac{x^2}{4\pi^2} = \frac{x^2}{4\pi}$.

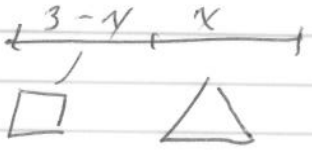
So the total area as a function of x is

$$A(x) = \left(\frac{2-x}{4}\right)^2 + \frac{x^2}{4\pi}.$$

(The answer is different if you make the x length into square and $2-x$ into circle - it is

$A(x) = \left(\frac{x}{4}\right)^2 + \frac{(2-x)^2}{4\pi}$ = area.)

3. You cut a 3 meter piece of wire into two pieces, One is a square and one is an equilateral triangle. What is total area enclosed.



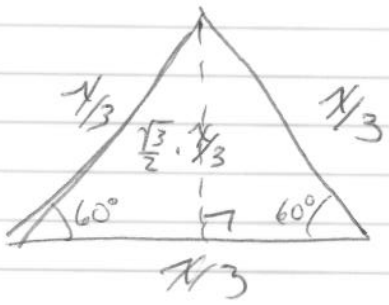
Answer: Let x = length made into triangle so

$3-x$ = length made into square.

Area of square is $\left(\frac{3-x}{4}\right)^2$

For the equilateral triangle

Each side is length $x/3$



so the height is $\frac{\sqrt{3}}{2}\cdot\left(\frac{x}{3}\right)$

(This is a fact about equilateral triangles)

So the area is

$$\tfrac{1}{2}\,\text{base}\cdot\text{height} = \tfrac{1}{2}\left(\tfrac{x}{3}\right)\cdot\left(\tfrac{\sqrt{3}}{2}\,\tfrac{x}{3}\right)$$

$$= \frac{\sqrt{3}\,x^2}{36}.$$

So the total area is

$$A(x) = \left(\frac{3-x}{4}\right)^2 + \frac{\sqrt{3}\,x^2}{36}.$$

The height of the equilateral triangle you can remember the "standard" size