Solutions H.W.2:

1) $f(x) = x^2 + 3x - 3$

a) Roots: We solve $0 = x^2 + 3x - 3$

so

$$x = \frac{-3 \pm \sqrt{9 - 4(-3)}}{2}$$

$$x = \frac{-3 \pm \sqrt{9 + 12}}{2} = \frac{-3 \pm \sqrt{21}}{2} = \begin{cases} \frac{-3 + \sqrt{21}}{2} \\ \frac{-3 - \sqrt{21}}{2} \end{cases}$$

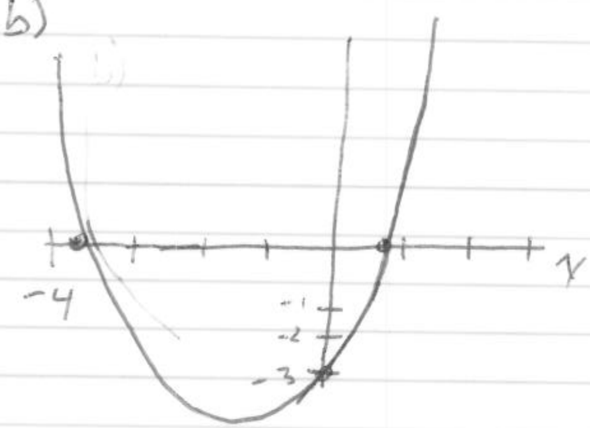
Now $4 < \sqrt{21} < 5$ and so we use

$\sqrt{21} \approx 4.5$ as a rough approximation

So $\frac{-3 + \sqrt{21}}{2} \approx \frac{1.5}{2} \approx .75$

$\frac{-3 - \sqrt{21}}{2} \approx -\frac{7.5}{2} \approx -3.75$

b)



c) Note $f(0) = -3$

Near $x = 0$ $f(x) \approx 3x - 3$

For $|x|$ large $f(x)$ is very large positive because of the x^2 term

2.) $g(x) = \dfrac{2x^2-8}{x+1}$

a) Solve $\dfrac{2x^2-8}{x+1} = 0$

or

$2x^2 - 8 = 0$

or

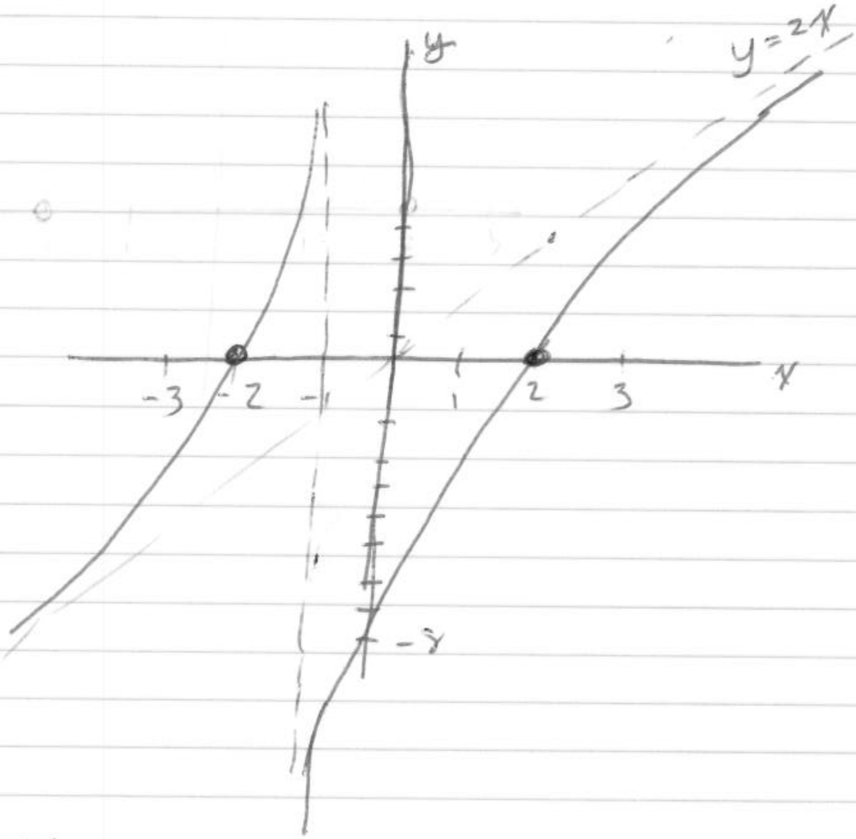
$2x^2 = 8$

or

$x^2 = 4$

$x = \pm 2.$

b)



c) Well $g(2) = g(-2) = 0.$

$g(0) = -8.$

When $x \approx -1$ then $2x^2 - 8 \approx -6$ and $x+1$ is close to 0

For x close to -1 but slightly bigger, $x+1 > 0$ so

$\dfrac{2x^2-8}{x+1} \approx \dfrac{-6}{\text{small positive}}$ = large negative

For x close to -1 but slightly to the left of -1,
$x+1$ is negative so $\frac{2x^2-8}{x+1} \approx \frac{-6}{\text{small negative}} = \text{large positive}$.

For large $|x|$

$$\frac{2x^2-8}{x+1} \approx \frac{2x^2}{x} \approx 2x$$

So the graph is close to the line $y = 2x$

3.) $h(x) = \frac{x^2-9}{x+3}$

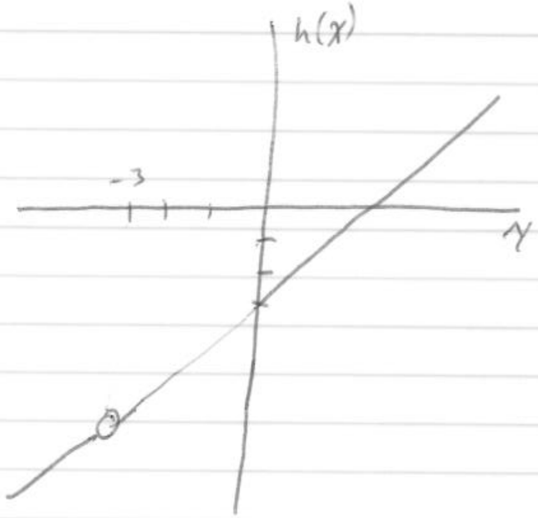
a). Roots are where $x^2 - 9 = 0$

so $x^2 = 9$

$x = \pm 3$.

(b. and c) Note $x^2 - 9 = (x+3)(x-3)$

so $\frac{x^2-9}{x+3} = \frac{(x+3)(x-3)}{x+3} = x-3$ when $x \neq -3$



We can not let $x = -3$ because the denominator is 0.

Problem 4-6 refer to graphs below.

MA 119 HOME WORK 2: Due Thurs. Sept. 17

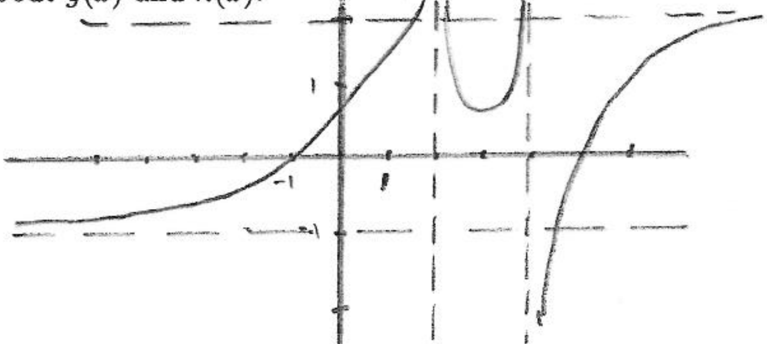
For each of the following,

a. Find the roots.

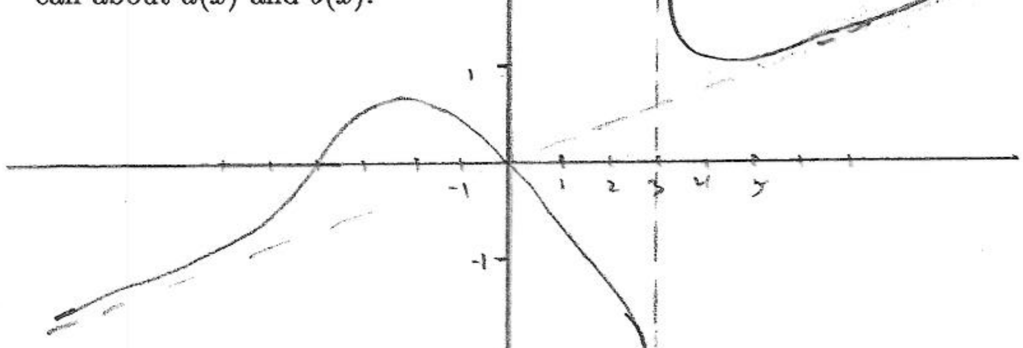
b. sketch the graph on a scale that shows all the interesting behavior of the function.

c. Comment briefly on how you figured out what the graph looks like (plot only a few points, otherwise, use information from the formula.)

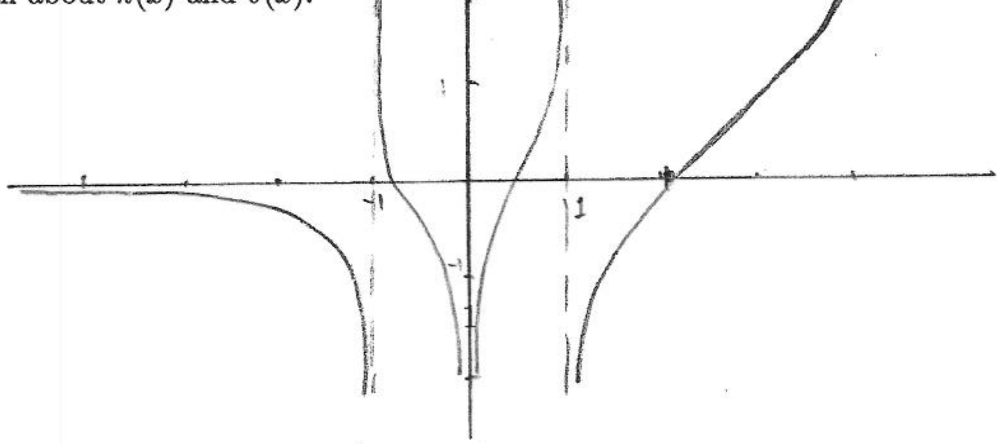
1. $f(x) = x^2 + 3x - 3$
2. $g(x) = (2x^2 - 8)/(x + 1)$
3. $h(x) = (x^2 - 9)/(x + 3)$
4. The figure below is the graph of a function $f(x) = g(x)/h(x)$. Say everything you can about $g(x)$ and $h(x)$.



5. The figure below is the graph of a function $y = a(x)/b(x)$. Say everything you can about $a(x)$ and $b(x)$.



6. The figure below is the graph of a function $y = k(x)/v(x)$. Say everything you can about $k(x)$ and $v(x)$.



4) For this graph.

$$f(x) = \frac{g(x)}{h(x)}.$$

When x is large positive, $f(x) \approx 2$

so $\frac{g(x)}{h(x)} \approx 2$

or

$g(x) \approx 2h(x)$ or $g(x)$ is approximately twice as large as $h(x)$.

For $x = 5$, $f(5) = 0$, so $g(5) = 0$.

For x near 4, $h(x)$ must be much closer to 0 than g.

and the same for x near 2.

For $x = -1$, $f(-1) = 0$ so $g(-1) = 0$.

For x very negative $\frac{g(x)}{h(x)} \approx -1$

so

$g(x) \approx -h(x)$.

Finally $g(x)$ and $h(x)$ are the same sign for $x > 5$, $2 < x < 4$ and $-1 < x < 2$,

and they are different sign at other x values.

5) Well $\frac{a(0)}{b(0)} = 0$, so $a(0) = 0$.

Also $\frac{a(-4)}{b(-4)} = 0$ so $a(-4) = 0$.

for x large positive and negative

$$\frac{a(x)}{b(x)} \approx 6x$$

So

$$a(x) \approx 6x \cdot b(x).$$

Near $x = 3$ we must have

$b(x)$ much closer to 0 than $a(x)$.

Finally, $a(x)$ and $b(x)$ are the same sign for $x > 3$, $-4 < x < 0$. They are different sign otherwise.

6). Well,

$k(x) = 0$ for $x = 2$ and $x \approx 1/2$ and $x \approx -3/4$.

$k(x)$ is much larger than $v(x)$ for x near 0 and x near $+1$ and x near -1.

For x very negative, $k(x)$ is much smaller than $v(x)$

For x very positive, $k(x)$ is much larger than $v(x)$.

Finally $k(x)$ and $v(x)$ are the same sign for $x > 2$, $\frac{1}{2} < x < 1$, $-1 < x < \frac{3}{4}$. they are different signs at other x values.