Solutions H.W. # 3 --

1. a) Find the roots of

$$f(x) = \frac{2x-6}{x^2+x-2}.$$

These occur where $2x-6=0$
or $x=3$.

c and b) To sketch the graph we note that points where the denominator is zero are important.

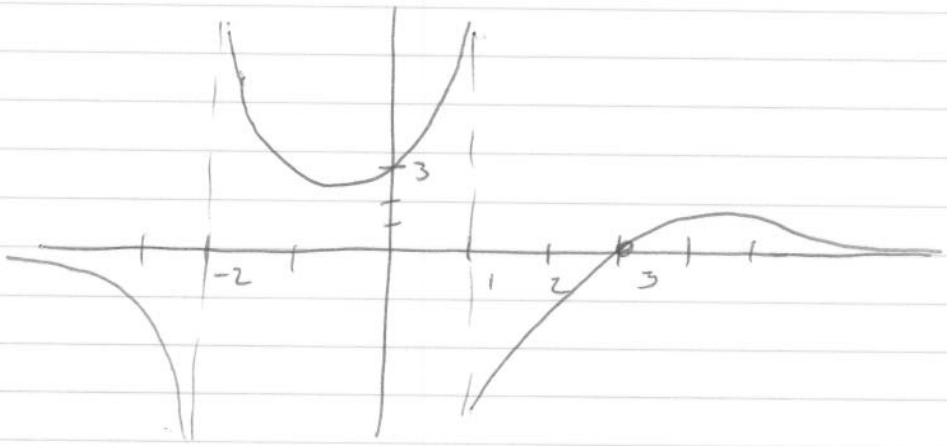
$$x^2+x-2=0$$

where

$$x = \frac{-1 \pm \sqrt{1-4\cdot(-2)}}{2} = \frac{-1\pm\sqrt{9}}{2} = \frac{-1\pm 3}{2} = \begin{cases} -4/2 = -2 \\ 2/2 = 1 \end{cases}$$

Vertical asymptotes occur at these points

Now $2x-6>0$ when $x>3$
$x^2+x-2>0$ when $x>1$ or $x<-2$



When $|x|$ is large $f(x) \sim \frac{2x}{x^2} \sim \frac{2}{x}$

So $f(x) \to 0$ as $x \to \pm\infty$.

2a) Find the roots of

$$g(x) = \frac{3-x^2}{x^2+2x+1}$$ There occur where $3-x^2=0$ or $x = \pm\sqrt{3}$.

(b and c) The denominator is zero (vertical asymptote)

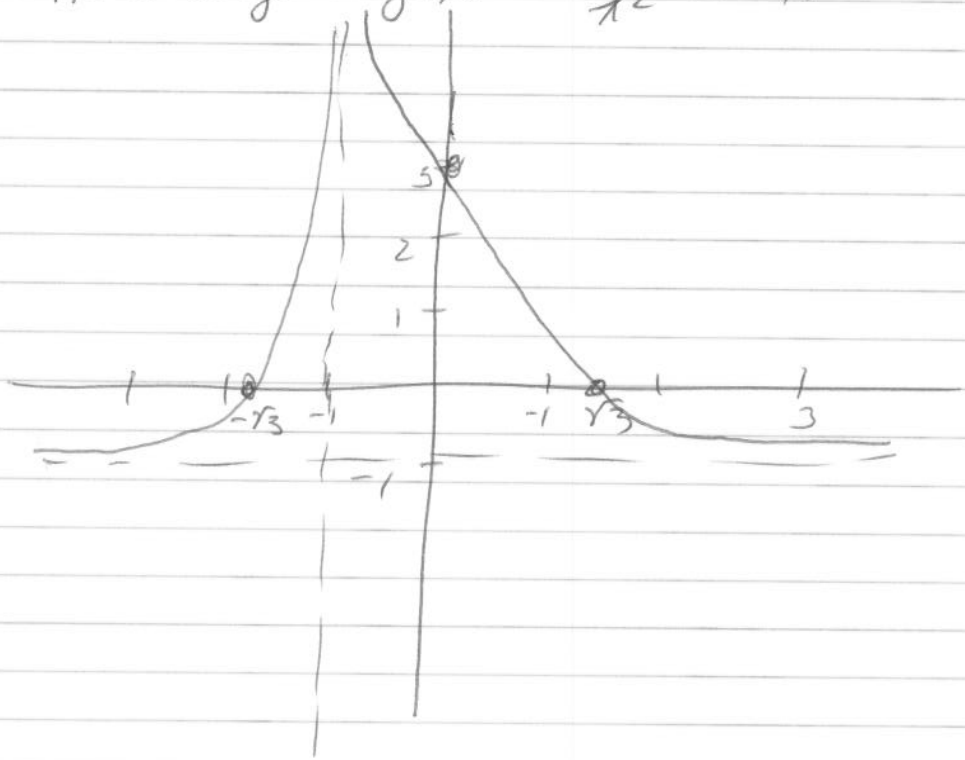
at $x^2+2x+1=0$

$$x = \frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2}{2} = -1$$

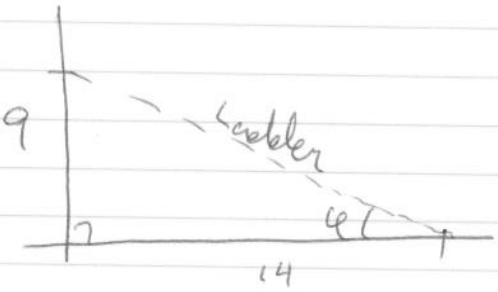
So $x^2+2x+1 \geq 0$ for all x,

and $3-x^2 > 0$ only for $|x| < \sqrt{3}$.

When $|x|$ is large $g(x) \sim \frac{-x^2}{x^2} = -1$.



3)



The ladder must be $\sqrt{9^2+14^2}$

$= \sqrt{81+196}$

$= \sqrt{277}$ ft.

So a) $\sin\theta = \frac{9}{\sqrt{277}}$

b) $\cos\theta = \frac{14}{\sqrt{277}}$

c) $\tan\theta = 9/14$

d) $\theta = \arctan(9/14) = \arcsin\left(\frac{9}{\sqrt{277}}\right) = \arccos\left(\frac{14}{\sqrt{277}}\right)$.

4) The ladder slides down at rate of 1 ft/min.

So if t is time in minutes,

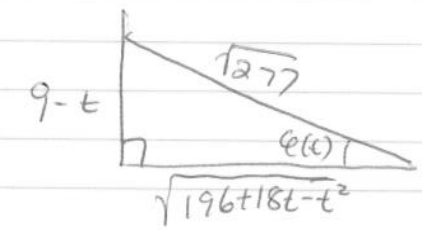
a) the height $h(t)$ = height at time $t = 9-t$

(choosing time 0 so that $h(0)=9$).

The ladder stay at length $\sqrt{277}$, so the distance from the wall is $\sqrt{\sqrt{277}^2-(9-t)^2}$

$= \sqrt{277-81+18t-t^2}$

$= \sqrt{196+18t-t^2}$.



b)

So the angle $\theta(t)$ satisfies $\sin(\theta(t)) = \frac{9-t}{\sqrt{277}}$

c) so $\theta(t) = \arcsin\left(\frac{9-t}{\sqrt{277}}\right)$.

5) Well

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - 1.$$

(NOTE: "Think" on assignment!

$\tan^2\theta = \sec^2\theta - 1$ not $-\sec^2\theta + 1$.)