NAME Key

Show your work and BE NEAT and organized.

1. Find the equation of the line through the points $(1, 2)$ and $(4, 11)$.

$\text{Slope} = \frac{11-2}{4-1} = \frac{9}{3} = 3$

$y = 3x + b$

Plug in $(1,2)$

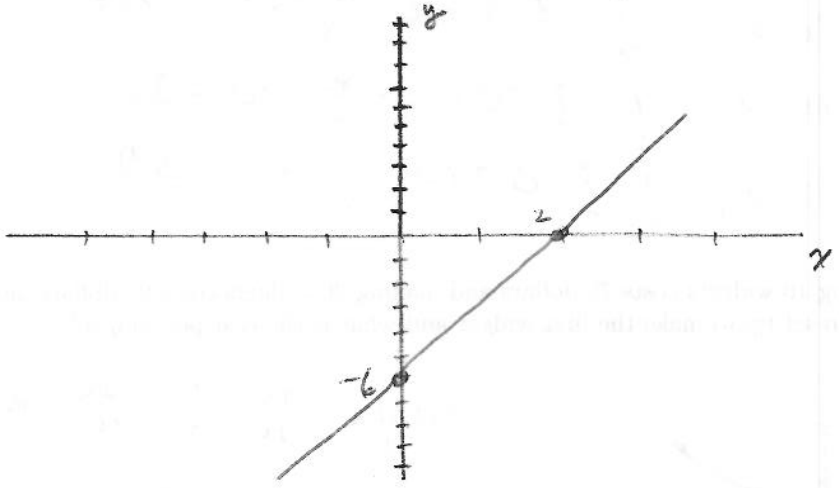
$2 = 3 \cdot 1 + b$

$b = -1$

So $y = 3x - 1$

(check $2 = 3 \cdot 1 - 1$, $11 = 3 \cdot 4 - 1$.)

2. Sketch the graph of the line with slope 3 and vertical intercept -6.



3. Find the solution set of

$$3x + 9 < 21.$$

Well $3x + 9 < 21$

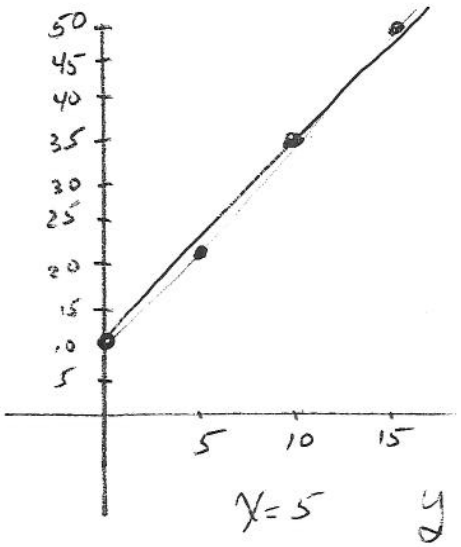
$3x + 9 - 9 < 21 - 9$

$3x < 12$

$x < \frac{12}{3}$

$x < 4$

4. For the data in the graph below, find a linear function in the form $y = mx + b$ which fits the data pretty well.



Points are close to a line

slope $\approx \frac{50-10}{15} = \frac{40}{15} = \frac{8}{3}$

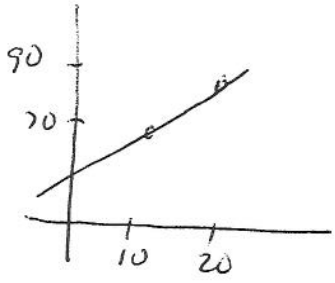
So $y = \frac{8}{3}x + 10$

$x = 5$ $\quad y = \frac{8}{3} \cdot 5 + 10 = \frac{40}{3} + 10 = 23\frac{1}{3}$

$x = 10$ $\quad y = \frac{8}{3} \cdot 10 + 10 = \frac{80}{3} + 10 = 36\frac{2}{3}$

$x = 15$ $\quad y = \frac{8}{3} \cdot 15 + 10 = \frac{120}{3} + 10 = 50.$

5. If making 10 widgets costs 70 dollars and making 20 widgets costs 90 dollars, how much does it cost to set up to make the first widget and what is the cost per widget?



x = # of widgets

y = cost

slope $= \frac{90-70}{20-10} = \frac{20}{10} = 2$

So $y = 2x + b$

Plug in $(10, 70)$

$70 = 2 \cdot 10 + b$

$70 = 20 + b$

$\Rightarrow b = 50$

So $y = 2x + 50$

Cost of setting up is \$50 ($x = 0$)

Cost per widget is \$2 (slope)

6. Find the roots and the vertex of

$$y = x^2 - 5x + 6.$$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 6}}{2}$

$x = \frac{5 \pm \sqrt{25 - 24}}{2}$

$x = \frac{5 \pm \sqrt{1}}{2}$

$x = \frac{5+1}{2} = \frac{6}{2} = 3$

or

$x = \frac{5-1}{2} = \frac{4}{2} = 2.$

roots 2, 3

7. If the profit from making x widgets is given by

$$p(x) = -x^2 + 8x + 3$$

what is the maximum profit that can be made?

Vertex occurs at $x = \frac{-b}{2a} = \frac{-8}{2(-1)} = 4$

$P(4) = -(4^2) + 8 \cdot 4 + 3 = -16 + 32 + 3 = 19$

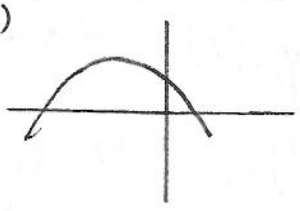
So max profit is 19

8. Which of the graphs below is the graph of

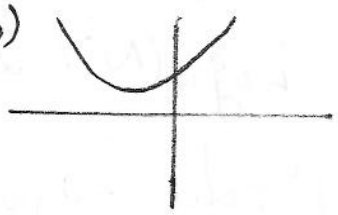
$$y = -x^2 + 3x + 2.$$

Explain how you made your choice in a sentence or two.

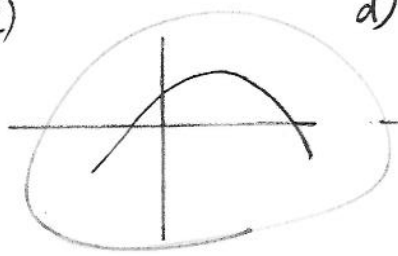
a)



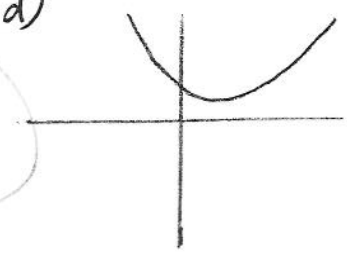
b)



c)



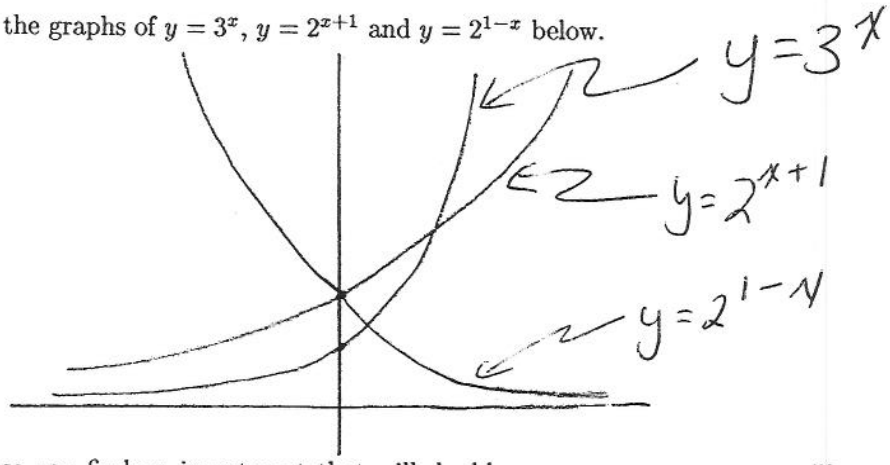
d)



The $-x^2$ implies downward concavity (∩) the y intercept is 2 and near $x = 0$ the graph is close to $3x + 2$ so has positive slope.

So C

9. Label the graphs of $y = 3^x$, $y = 2^{x+1}$ and $y = 2^{1-x}$ below.



10. Suppose you find an investment that will double your money every year. If you invest 100 dollars, how much will you make during the 4th year?

100 at start
200 at end of year 1
400 at end of year 2
800 at end of year 3
1600 at end of year 4

So you make 800 dollars during 4th year.

11. The number of digits need to write an integer n is approximately $\text{Log}_{10}(n)+1$ (it takes 2 digits to write 10, 3 digits to write 100 and so on). If a particular N takes 25 digits to write, how many digits will N^2 take?

Well $\text{Log}_{10}(N)+1 = 25$

So $\text{Log}_{10}(N) = 24$

So $\text{Log}_{10}(N^2) = 2\text{Log}_{10}(N) = 48$

So N^2 take $\text{Log}_{10}(N^2)+1 = 49$ digets

12. Solve for x in

$$\text{Log}_{10}(3x) - \text{Log}_{10}(x+2) = \text{Log}_{10}(2).$$

well $\text{Log}_{10}\left(\frac{3x}{x+2}\right) = \text{Log}_{10}(2)$

So $\frac{3x}{x+2} = 2$ So $x = 4$

So $3x = 2(x+2)$

or $3x = 2x+4$

$3x - 2x = 4$

13. Solve for x in

$$4^x = 8^2.$$

$4^x = (2^2)^x = 2^{2x}$

$8^2 = (2^3)^2 = 2^6$

So $2^{2x} = 2^6$

So $\text{Log}_2(2^{2x}) = \text{Log}_2(2^6)$

$2x = 6$ or $x = 3$

14. The Richter scale for earthquake of "intensity" i is given by

$$R(i) = \text{Log}_{10}\left(\frac{i}{i_0}\right),$$

where i_0 is the envergy of the smalles earthquake that can be detected.

How much more powerful is an earthquake with $R = 9$ than an earthquake with $R = 7$?

If $R(i) = 9$ then

$9 = \text{Log}_{10}\left(\frac{i}{i_0}\right)$

So $10^9 = i/i_0$

So $10^9 \cdot i_0 = i$

If $R(i) = 7$ then

$7 = \text{Log}_{10}(i/i_0)$

$10^7 = i/i_0$

So $10^7 i_0 = i$

So a 9 earthquake is 100 times more intense than a 7 earthquake because

$$\frac{10^9 i_0}{10^7 i_0} = \frac{10^9}{10^7} = 10^{9-7} = 100$$