Show all work and justify all answers. Neatness definitely counts.

1. Compute

\[ \int x e^{x^2} \, dx \]

Let \( u = x^2 \), \( \frac{du}{dx} = 2x \), \( \frac{1}{2} \, du = x \, dx \)

\[ = \int e^u \left( \frac{du}{2} \right) \]
\[ = \frac{1}{2} \int e^u \, du \]
\[ = \frac{1}{2} e^u + C \]
\[ = \frac{1}{2} e^{x^2} + C \]

(check: \( \frac{1}{2} e^{x^2} + C \)')
\[ = \frac{1}{2} \cdot 2xe^{x^2} + 0 \]
\[ = xe^{x^2} \]

2. Compute

\[ \int x e^x \, dx \]

Let \( u = x \), \( \frac{du}{dx} = 1 \), \( v = e^x \)

\[ = xe^x - \int e^x \, dx \]
\[ = xe^x - e^x + C \]

(check: \( (xe^x - e^x + C)' \))
\[ = e^x + xe^x - e^x + 0 \]
\[ = xe^x \]

3. Compute

\[ \int \frac{x}{x + 3} \, dx \]

Let \( u = x + 3 \), \( \frac{du}{dx} = 1 \)
\[ du = dx \]
\[ \frac{u - 3}{u} \, du \]
\[ = \int \frac{u - 3}{u} \, du \]
\[ = \int \left( 1 - \frac{3}{u} \right) \, du \]
\[ = u - 3 \ln(u) + C \]
\[ = (x + 3) - 3 \ln(x + 3) + C \]

(check: \( (x + 3) - 3 \ln(x + 3) + C \)')
\[ = 1 - \frac{3}{x + 3} + 0 \]
\[ = \frac{x + 3 - 3 \ln(x + 3)}{x + 3} \]
\[ = \frac{x + 3}{x + 3} - \frac{3}{x + 3} \ln(x + 3) \]
4. Compute
\[ \int_1^2 (x - 1)^5 \, dx \]
Let \( u = x - 1 \)
\[ du = dx \]
\[ u = 0 \quad \text{if} \quad x = 1 \]
\[ u = 1 \quad \text{if} \quad x = 2 \]
\[ \int_0^1 u^5 \, du = \frac{u^6}{6} \bigg|_0^1 = \frac{1}{6} - 0 = \frac{1}{6}. \]

5. Compute
\[ \int_0^1 (x^2 - 3)^2 \, dx \]
\[ = \int_0^1 (x^4 - 6x^2 + 9) \, dx \]
\[ = \frac{x^5}{5} - \frac{6x^3}{3} + 9x \bigg|_0^1 \]
\[ = \frac{1}{5} - 2 + 9 \]
\[ = 7\frac{1}{5} = \frac{36}{5}. \]
6. Compute the total area trapped between the graphs of \( y = x^3 \) and \( y = 4x \). (Show all work.)

\[
y = x^3 \text{ and } y = 4x
\]
intersect when
\[4x = x^3\]
So \( x = 0 \) or \( 4 = x^2 \) i.e. \( x = \pm 2\).

So 3 points of intersection

So the area we want is
\[
\int_{-2}^{0} x^3 - 4x \, dx + \int_{0}^{2} 4x - x^3 \, dx
\]
\[
= \left[ \frac{x^4}{4} - \frac{4x^2}{2} \right]_{-2}^{0} + \left[ 4x^2 - \frac{x^4}{4} \right]_{0}^{2}
\]
\[
= 0 - \left( \frac{16}{4} - 2(2^2) \right) + \left( 4(2)^2 - \frac{2^4}{4} \right) - 0
\]
\[
= -\left( \frac{16}{4} - 8 \right) + \left( 8 - \frac{16}{4} \right)
\]
\[
= -(4 - 8) + (8 - 4)
\]
\[
= 4 + 4 = 8.
\]

So the total area is 8.
7. The velocity \( v(t) \), where \( t \) is time in hours, of a car traveling down a road is pictured below. Find how far the car travels between time \( t = 0 \) and time \( t = 4 \) (show your work and justify your answer).

If \( p(t) \) = position then \( p'(t) = v(t) \)

So, by F.T.C.

\[
\int_0^4 v(t) \, dt = p(4) - p(0)
\]

But \( \int_0^4 v(t) \, dt = \text{area under graph} \)

\[
= 65
\]

So \( p(4) - p(0) = \text{distance travelled} \)

\[
= 65 \text{ mi.}
\]
8. Approximate the volume of a "square pyramid" (as pictured below) which is 6 meters on each side at the base and 6 meters from base to tip. (If you happen to remember the formula for the volume of this pyramid, DO NOT USE IT!, find a numerical approximation using ideas from class.)

\[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]

\[ \text{Monument} = 4 \times 4 \times 2 = 32 \]
\[ \text{Highest} = 2 \times 2 \times 2 = 8 \]

So total approximate volume is

\[ \frac{72}{32} + 8 \]
\[ 112 \text{ cubic meters.} \]

9. Is your estimate above too large or too small? How would you find an approximation that is on the other side of the actual volume?

This is too large because pyramid is inside boxes.

We could use boxes on the inside (2)

\[ \text{Volume} = 32 + 8 = 40 \]