The following is a collection of sample problems. It is NOT a study guide to give you an idea of the type and level of quiestions on the final. (E.g. If there isn't a question on a particular topic listed here it may still appear on the final.)

There will be some "memory" questions like the first page of the second midterm. You should review definitions and the main theorems. In addition to those topics covered in the second midterm, you should be prepared to comment on the basic ideas of calculus, the Fundamental Theorem and the basics of the numerical integration.

Other exercises:

1. $$\int_1^5 x^3 \ln x \, dx$$

2. $$\int_0^2 \frac{3x}{x+2} \, dx$$

3. If $f(x)$ has negative first derivative and positive second derivative on the interval $2 < x < 6$, what can you say about the trapazoid rule approximation for
$$\int_2^6 f(x) \, dx$$
Justify your answer.

4. Suppose $f(x)$ is positive, continuous and decreasing for all $x > 0$. Does this imply that
$$\int_1^\infty f(x) \, dx$$
converges?

5. Compute (if it converges)
$$\int_1^\infty e^{-x} \sin(x) \, dx$$

6. Is it true that the trapazoid rule is always MORE accurate than the Riemann sum using "left hand end points"? What can you say about the relative accuracy of these methods?

7. If you cut the step size in a trapazoid rule in half are you guarenteed of having 1/4th the error?

8. True of False (be careful)
$$\int_{-1}^1 \frac{1}{x} \, dx = 0$$
Justify your answer.

9. Verify that the volume of a sphere is $4\pi r^3/3$ (justify any formulas that you use).

10. Find the volume of pyramid that has an equilateral triangle as its base with side length 1 unit and tip 2 units above the base.

11. In the preceeding problem it was not specified that the tip of the pyramid is directly over the center of the base. Does this matter? Explain your answer.

12. Find the volume in common in the intersection at right angles of two circular cylindars of radius r.

13. Find the work necessary to fill an inverted cone with height 10m and radius 4m with water (water has mass 1000kg/m^3).

14. Determine if the following converge or diverge. If you can determine the value of the sum (if it converges) that will be extra credit.

$$\sum_{n=0}^{\infty} \frac{n!}{5^n}$$

$$\sum_{n=0}^{\infty} \frac{e^n}{n!}$$

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$\sum_{n=0}^{\infty} (n+1) \left(\frac{1}{3}\right)^n$$

15. Give the radius of convergence of

$$\sum_{n=0}^{\infty} \frac{x^n}{n^5 5^n}$$

$$\sum_{n=0}^{\infty} \frac{5^n x^n}{n^{10}}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n^n}$$

$$\sum_{n=0}^{\infty} \frac{n(x-4)^n}{n^3+1}$$

16. State the fifth degree Taylor polynomial centered at $x = 1$ of e^x.

17. What is the maximum difference betwee e^x and the polynomial in the preceeding problem on the interval $0 < x < 2$?

18. Find the third degree Taylor polynomial for the funciton $f(x) = 1/x$ centered at $x = 2$. What is the maximum error between this polynomial and f on the interval $1 < x < 3$?

19. Find the 4th degree Taylor polynomial centered at $t = 0$ for the function $y(t)$ that satisfies

$$\frac{dy}{dt} = -2y + t^2, \quad y(0) = 1$$

20. Find the 5th degree Taylor polynomial centered at $t = 0$ for the function $y(t)$ that satisfies

$$\frac{d^2y}{dt^2} + t\frac{dy}{dt} + ty = 0, \quad y(0) = 1, y'(0) = 2$$