The following are a couple of additional sample problems for the Laplace transform and non-linear systems section since the second exam. The homework problems (not to be handed in) are also excellent examples.

The final is "inclusive", so also look at the first and second exams, associated sample problems and homework problems.

1. For the initial value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 6y = 5\cos(2t), \quad y(0) = 2, y'(0) = 3$$

   (a) Compute the Laplace transform of the solution of this initial value problem.

   (b) Compute the poles of the Laplace transform you computed in part (a).

   (c) Describe the long-term behavior of the solution of the initial value problem (say what you can about the long-term behavior and state what the poles do not tell you about the long-term behavior of the solution).

2. Suppose the Laplace transform of $f(t)$ is

$$\frac{2}{1 - e^{-3s}}$$

   (a) What are the poles of this Laplace transform? (Hint: There are infinitely many poles.)

   (b) The poles give the types of functions that are the summands in $f(t)$, what are these functions?

3. What should

$$\mathcal{L}\left[\frac{d\delta_a}{dt}\right]$$

   equal?

4. (a) Consider the system

$$\begin{aligned} \frac{dx}{dt} &= -2x + y + 3x^2 + 3y^2 \\ \frac{dy}{dt} &= x + 3y + x^2 + y^2 \end{aligned}$$

   What is the linear system approximating this system near $x = y = 0$?

   (b) Describe the set of initial conditions which give solutions which tend to the origin as $t \to \infty$.

5. For the non-linear system

$$\begin{aligned} \frac{dx}{dt} &= x(1 - x) - 2xy \\ \frac{dy}{dt} &= y\left(1 - \frac{y}{2}\right) - xy \end{aligned}$$

   (a) What are the equilibrium points?

   (b) What are the linear approximations of the systems near the equilibrium points and what types of linear systems are they?

   (c) What does your answer in part (b) tell you about the non-linear system?