Following are some sample problems for the second exam, covering those sections of chapter 2 and 3 that we have covered–Including harmonic oscillators.

1. Say everything you can about the system

$$\frac{dY}{dt} = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} Y$$

   and its solutions.

2. Say everything you can about the solutions of the equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 8y = 0.$$

3. What types of linear systems can occur in the one parameter family

$$\frac{dY}{dt} = \begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix} Y$$

   if a can be any real number.

4. Can you choose the damping coefficient in the harmonic oscillator equation

$$\frac{d^2y}{dt^2} + b\frac{dy}{dt} + 5y = 0$$

   so that the period of the oscillations is 2?

5. For the system

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -x + (1 - x^2)y \end{aligned}$$

   (a) Find all the equilibrium points.

   (b) Sketch the direction field (I probably wouldn't ask you to do this on a test since it would take too long, I might give you the direction field and ask you to compute the vector field at the point $(2, 1)$.

   (c) From the direction field, sketch the solution with initial condition $(0.2, 0)$. Also sketch the $x(t)$ and $y(t)$ graphs.

6. For the system

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -\sin(x) - 0.2y \end{aligned}$$

   (a) Find the equilibrium points.

   (b) Sketch the direction field (Find the regions of the (x, y) plane where the direction field is going up-right, down-right, up-left and down-left.)

   (c) Sketch a solution with initial condition near $(0, 0)$.

(d) Suppose we approsimate $-\sin(x)$ with its tangent line at zero $-sin(x) \approx -x$. What can you say about the resulting (linear) system.

7. Two students are preparing for an exam in differential equations– the question they are doing is to describe the solutions of

$$\frac{dY}{dt} = \begin{bmatrix} 3 & 1 \\ -1 & -4 \end{bmatrix} Y$$

One of the students says "The $x(t)$ graph goes to infinity as t increases. The other says "No, the $x(t)$ graph goes to minus infinity as t increases." Who is right?

8. For an underdamped harmonic oscillator with mass $m = 1$, (so $b \neq 0$, but is small)) if we double the spring constant and double the damping coefficient, what, if anything, can we say about the natural period.