(Sample Homework Writeup)

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Discussion AD
Weds. 2-3

Assignment: 1.1: #1, 12
1.2: #26.

1.1, #1. Find the equilibrium solutions for

$$\frac{dy}{dt} = \frac{y+3}{1-y}.$$

Ans: Equilibria are constant solutions $y(t) = y_0$ so they occur at y_0 values where $\frac{dy}{dt} = 0$

So we solve $\frac{y+3}{1-y} = 0$

i.e.

$$y+3=0$$

So $y=-3$ is the only equilibrium solution.

1.1, #12. The velocity of a skydiver is modelled by

$$m\frac{dv}{dt} = mg - kv^2.$$

a. Assuming $v>0$ corresponds to downward velocity (so the model makes sense), we have $(m, g, k > 0)$

$$\frac{dv}{dt} = g - \frac{k}{m}v^2$$

Equilibrium points are

$$0 = g - \frac{k}{m} v^2$$

or

$$v^2 = \frac{m}{k} g$$

or

$$v = \pm \frac{\sqrt{mg}}{\sqrt{k}}.$$

Note $\frac{dv}{dt} > 0$ if $-\frac{\sqrt{mg}}{\sqrt{k}} < v < \frac{\sqrt{mg}}{\sqrt{k}}$

$\frac{dv}{dt} < 0$ if $v > \frac{\sqrt{mg}}{\sqrt{k}}$ or $v < -\frac{\sqrt{mg}}{\sqrt{k}}$

So $v(t)$ solution increases if $-\frac{\sqrt{mg}}{\sqrt{k}} < v < \frac{\sqrt{mg}}{\sqrt{k}}$

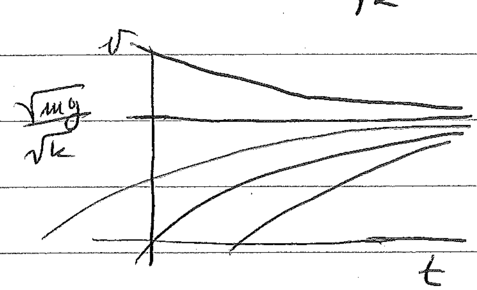
and decreases if $v > \frac{\sqrt{mg}}{\sqrt{k}}$.

(Note solutions also decrease if $v < -\frac{\sqrt{mg}}{\sqrt{k}}$ but this is not "physical" so we restrict to $v \geq 0$)

All solutions with $v(0) \geq 0$ approach $\frac{\sqrt{mg}}{\sqrt{k}}$ as $t \to \infty$.

b) A skydiver starts with $v(0) > 0$ so velocity approaches the "terminal velocity" of

$$v = \frac{\sqrt{mg}}{\sqrt{k}} \text{ as } t \to \infty.$$



1.2, #26. Solve the IVP

$$\frac{dy}{dt} = ty, \quad y(0) = 3$$

Well, this is separable so we write,

$$\frac{1}{y}\frac{dy}{dt} = t$$

or

$$\int \frac{1}{y}\frac{dy}{dt}\,dt = \int t\,dt$$

$$\int \frac{1}{y}\,dy = t^2/2 + c$$

so

$$\ln(|y|) = t^2/2 + c$$

Solving for y gives

$$|y| = e^{t^2/2 + c} = e^{t^2/2} e^c$$

Allowing k to be either + or −, we have

$$y(t) = k e^{t^2/2}$$ as the general solution

Using $y(0) = 3 = k e^{0^2/2} = k \cdot 1$

we have the particular solution $y(t) = 3e^{t^2/2}$

(Check: $y(0) = 3e^0 = 3$ and

$$\frac{dy}{dt} = \frac{d}{dt}(3e^{t^2/2}) = 3e^{t^2/2} \cdot \frac{2t}{2} = t \cdot 3e^{t^2/2} = ty.$$

So the solution checks —)