These are sample problems like those that will be on the exam. For more samples, see the exercise sets in Sections 1.1-1.8 and the Review problems from chapter 1.

1. Find the general soluiton and the solution with $y(0) = 2$ for

$$\frac{dy}{dt} = y^2(t + 6).$$

2. Find the genearl solution of

$$\frac{dy}{dt} = -2y - 3\cos(3t).$$

3. Suppose you know that $dy/dt = f(y)$ has exactly three equiliabrium points. The are sinks at $y = -1$ and $y = 3$ and a source at $y = 1$.

   (a) Sketch a continuous function $f(y)$ for which $dy/dt = f(y)$ would have this behavior.

   (b) Describe the behavior of the solution with $y(0) = 0$.

4. Suppose you have a ten gallon bucket full of clean water. You pour soda containing 96 teaspoons of sugar per gallon into the bucket at a rate of 1/4 gallon per minute. Suppose you also pour 1/4 gallon of pure water into the bucket per minute. The water is well mixed and you remove 1/2 gallon/minute out the bottom (so the volume stays constant).

   (a) Give a model for the number of teaspoons of sugar in the bucket at time t. Be sure to name your variables and give units.

   (b) What is the long-term behavior of the amount of sugar in the bucket?

   (c) How much sugar is in the bucket at time $t = 5$?

5. (a) Compute the first 2 steps of Euler's method with $\Delta t = 1$ $y(0) = 3$ for

$$\frac{dy}{dt} = y^2 + y - 6t.$$

   (b) Repeat the problem above with $y(2) = 3$.

6. For

$$\frac{dy}{dt} = -y^3 + 3y + a$$

   sketch the phase line that occur for different a values and give the bifurcation values of a.

7. Suppose you buy a condominium for 150,000 dollars. You borrow 120,000 at 5 percent interest, compounded continuously and you make payments of 200 dollars per week (so you may consider payments to be made continuously as well).

   (a) Give a differential equations model for the amount of money you owe at time t where $t = 0$ is when you take out the mortgage. Be sure to label all your variables.

   (b) When will you pay off the mortgage?

8. Say everything you can about solutions of

$$\frac{dy}{dt} = -2y + b(t)$$

   with $b(t)$ satisfying $0 < b(t) < 6$ for all t.