SNOW DAY CLASS

Thanks to the wonders of technology, we can still have a class on the snow day...While the number of classes is smaller, the required material is not, so we'll try out a bit of the "flipped" class philosophy.

Daily Problem For Friday: Bring today's "Daily" (check to make sure your answer is correct and you have followed the instruction–see below) AND FOR FRIDAY Section1.2: number 17 (THIS MUST BE ON A SEPARATE PIECE OF PAPER!)
Weekly Assignment Due Next Weds: Section 1.1: 6,7,8,9,11,20, Section 1.2: 5,11,17,34,42
May add a vew more on Friday.

First, some comments about the Daily problem you handed in last time:

1. Remember, hand in one sheet of paper with writing on one side. This means you will have to think carefully about what you need to say to do the problem and say it efficiently. Write in sentences, do not crowd and be neat. It is best to set off equations on their own lines (that forces blank space and makes the paper much more readable).

2. Follow the homework instructions at math.bu.edu/people/rockford

3. Question: Is $L(t) = 2t - 2L_0$ a solution of
$$\frac{dL}{dt} = 2(1 - L)?$$
NO! Just check
$$\frac{d(2t - 2L_0)}{dt} = 2 - 0$$
and
$$2(1 - (2t - 2L_0)) = 2 - 4t + 4L_0$$
and these are not equal for all t. Check all solutions!!

4. I posted a solution to the Daily problem on the course home page–Definitely check this out, both for content and for form. Many of you found (or tried to find) the solution and this is unnecessay for solving the problem.

5. When asked "how should I prepare for the exams" the first thing I'll say is "make sure you understand and can do all the homework problems"...

While I pretty much gave everyone who handed anything in credit for this assignment, from Friday on, I will not accept sloppy work or work that does not follow the homework instructions.

Now for more separable equations: First a warm up

$$\frac{dy}{dt} = 2ty + y$$

This is separable ($2ty + y = (2t+1)y$) so we can separate and write

$$\frac{1}{y}\frac{dy}{dt} = 2t + 1$$

so

$$\int \frac{1}{y}\frac{dy}{dt}\, dt = \int 2t + 1\, dt$$

or

$$\int \frac{1}{y}\, dy\, dt = \int 2t + 1\, dt$$

so

$$\ln(|y|) = t^2 + t + c.$$

Solving for $y(t)$ gives

$$|y(t)| = e^{t^2+t+c} = e^{t^2+t}e^c$$

Now both the left hand side and the right hand side are always positive. We can get rid of the absolute value signs on the left by replacing e^c with a constant k which can have any value (positive or negative) and

$$y(t) = ke^{t^2+t}.$$

Of course, the next step is to check by pluggin back into the original equation and making sure the left equals the right sides for all t.

We can find a value for k if we are given a value for $y(t_0)$ at some value t_0. For example, if we are given $y(1) = -3$ then

$$-3 = y(1) = ke^{1^2+1} = ke^2$$

so $k = -3/e^2 = -3e^{-2}$ and the particular solution of the initial value problem (impressive vocabulary) is

$$y(t) = -3e^{-2}e^{t^2+t}.$$

Here's another example that shows even easy looking equations can have odd and interesting properties:

$$\frac{dy}{dt} = y^2$$

This is separable. In face (VOCABULARY) it is "autonomous"–there are no t's explicitly in the equation (they are implicitly present since y is a function of t, but the rules for how things change don't depend on t.)

So

$$\int \frac{1}{y^2}\frac{dy}{dt}\,dt = \int 1\,dt$$

and

$$\int \frac{1}{y^2}\,dy = t + c$$

so

$$\frac{-1}{y} = t + c$$

and

$$y(t) = \frac{-1}{t+c}$$

.

If we are given that $y(0) = 2$ for example, then

$$2 = y(0) = \frac{-1}{0+c}$$

and $c = -1/2$, so the particular solution is

$$y(t) = \frac{-1}{t-(1/2)} = \frac{1}{(1/2)-t}$$

This funciton "blows up " (has a vertical asymptote) at $t = 1/2$. In the old days, you would say that the solution exists for asll $t \neq 1/2$. Now it is more common, particularly when thinking of t as time, to say that the solution exists for $t < 1/2$ and it is a different solution for $t > 1/2$–this actually makes more sense– nobody comes back from infinity (e.g., if y is the voltage, then when y goes to infinity, the machine melts and doesn't just magically restart afte r$t = 1/2$).

When you recognize that an equation is separable, it gives you at least a start...but you have to remember that you still have to do 2 integrals and some algebra to solve for $y(t)$ and we all know that integrals and algebra can be hard (or even impossible!)...Once you are done, you may have an equation that is so complicated that figuring out what it does is still hard...so even in cases where a formula for a solution might be obtainable, it isn't automatically the best way to approach the problem...

There are lots of "standard" types of story problems that can be done with this method. Here is one...

MIXING PROBLEM: Suppose road sald is polluting a small pond. The pond has a volume of 10,000 cubic meters. Two streams flow into the pond, on carrying 20 cubic meters per day of water into the pond of water with 1 kg/cubic meter of road salt and the other carrying 30 cubic meters per day of water into the pond with 0.5 kg/cubic meter of road salt. Suppose

50 cubic meters per day of water flow out a third stream, so the volume in the pond stays constant.

Question: What is the concentration of road salt in the pond?

Answer: Let t = time in days and $S(t)$ = kg of salt in the pond at time t. The story tells us how S changes with time.

$$\frac{dS}{dt} = \text{rate salt in via two streams and out via third}$$

Here $20 \cdot 1 = 20$ kg salt per day comes in via the first stream, $30 \cdot 0.5 = 15$ kg salt per day comes in via the second stream. The amount leaving via the third stream is $50\cdot$ concentration of salt in the pond, which is $S/10000$, so our differential equation is

$$\frac{dS}{dt} = 20 + 15 - 50 \cdot \frac{S}{10000}$$

or

$$\frac{dS}{dt} = 35 - \frac{S}{200}$$

This equation is separable, so we can solve...but do we really care down to the last gram how much salt is in the pond? No...we haven't even been told what $S(0)$ is...so let's just think instead of compute.

If S is small then $dS/dt > 0$, if S is big then $dS/dt < 0$. If $S = 35 \cdot 200 = 7000$ then $dS/dt = 0$ so this is an equilibrium point and all solutions approoach this value as t increases. Something we might want to know is "how long" does it take for S to get close to 7000? This we can determine via solving...go ahead. but we know the concentration approaches $7000/10000 = 7/10$ kg per cubic meter as t increases.

COMMENT: NOTICE, we asked about concentration but we set up the equation using total salt in kg. This is because it is easy to set up the equation using total salt and rather tricky using concentration (try it!). Once you have $S(t)$ it is easy to find the concentration– another example where choosing the right variables at the start makes a big difference.

Have fun in the snow, see you Friday.