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MR2114679 (2005m:11105) 11G05 (11R23)
Pollack, Robert [Pollack, Robert[2]] (1-BOST)
An algebraic version of a theorem of Kurihara. (English summary)
J. Number Theory **110** (2005), *no. 1,* 164–177.

In this paper, the author proves a result regarding the size and structure of the p-primary part of the Shafarevich-Tate groups $Ш(E/\mathbb{Q}_n)$ of an elliptic curve $E/\mathbb{Q}$, where $\mathbb{Q}_n$ is the n-th layer of the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$ and p is an odd prime of supersingular reduction. The author's conclusion about the size of the p-primary part $Ш(E/\mathbb{Q}_n)[p^\infty]$ is identical to an earlier result of M. Kurihara [Invent. Math. **149** (2002), no. 1, 195–224; MR1914621 (2003f:11078)]. However, the author provides a completely algebraic argument. The author assumes that $E(\mathbb{Q})$ is finite, p does not divide the Tamagawa factors $\mathrm{Tam}(E/\mathbb{Q})$, and $Ш(E/\mathbb{Q})$ has no p-torsion. These hypotheses are weaker than the analytic hypotheses in Kurihara's work, though the two sets of hypotheses would be equivalent under the Birch and Swinnerton-Dyer conjecture. Under the above assumptions, the author proves that $E(\mathbb{Q}_n)$ is finite, and he provides a precise formula for the order of $Ш(E/\mathbb{Q}_n)[p^\infty]$ for all $n \geq 0$.

The author starts with a review of the failure of Mazur's control theorem for the Selmer group $\mathrm{Sel}_p(E/\mathbb{Q}_n)$ at a supersingular prime. He shows that the size and structure of this Selmer group can be determined by studying a quotient of the Iwasawa algebra by the image of the local points of the formal group $\widehat{E}$ attached to E. In order to study this quotient, he exploits the trace-compatible local points of $\widehat{E}$ considered by S. Kobayashi [Invent. Math. **152** (2003), no. 1, 1–36; MR1965358 (2004b:11153)]. The author facilitates the computation of the order of the quotient by introducing two types of invariants, which are of similar flavor to the classical μ and λ invariants. Once the size and structure of $\mathrm{Sel}_p(E/\mathbb{Q}_n)$ are thus determined, one can draw the conclusions for $E(\mathbb{Q}_n)$ and $Ш(E/\mathbb{Q}_n)[p^\infty]$.

Reviewed by *Anupam Saikia*

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