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Kida's formula and congruences. (English summary)
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A Kida-type formula (an analogue for Iwasawa λ-invariants of the Riemann-Hurwitz genus formula) is proved for the Selmer groups of a rather general class of p-adic representations. Let F be a number field which is totally real or totally imaginary, p an odd prime number, V a nearly ordinary p-adic Galois representation of F (i.e. of $G_F = \mathrm{Gal}(\overline{F}/F)$) defined over a sufficiently large finite extension K of $\mathbb{Q}_p$, and T a G_F-stable $\mathcal{O}$-lattice in V, where $\mathcal{O}$ is the ring of integers of K. Set $A = V/T$. Then one can define the Selmer group $\mathrm{Sel}(F_\infty, A)$ of A over the cyclotomic $\mathbb{Z}_p$-extension F_∞ of F. It is a module over $\Lambda = \mathcal{O}[\![\mathrm{Gal}(F_\infty/F)]\!]$, and its algebraic Iwasawa invariants $\lambda(F_\infty, A)$ and $\mu(F_\infty, A)$ are defined. If F'/F is a finite extension and $F'_\infty = F'F_\infty$, then $\mathrm{Sel}(F'_\infty, A)$ and its invariants $\lambda(F'_\infty, A)$ and $\mu(F'_\infty, A)$ are defined similarly. The main result is: Theorem. Let F'/F be a finite Galois extension of a p-power degree. Assume that T satisfies some technical assumptions. If $\mathrm{Sel}(F_\infty, A)$ is Λ-cotorsion with algebraic μ-invariant zero, then so is $\mathrm{Sel}(F'_\infty, A)$. Moreover, in this case, one has $\lambda(F'_\infty, A) = [F'_\infty\colon F_\infty]\cdot\lambda(F_\infty, A) + \sum_{w'} m_{w'}(V)$. Here, the sum is over the places w' of F'_∞ which are prime to p and ramified in F'_∞/F_∞, and $m_{w'}(V)$ is a certain local invariant of V defined in terms of the behavior of V when twisted by characters of $\mathrm{Gal}(F'_{\infty,w'}/F_{\infty,w})$. If V is associated to a cuspidal (elliptic modular) eigenform f and F' is abelian over $\mathbb{Q}$, then the same results hold for the analytic Iwasawa invariants of f.

These results generalize previous works of Y. Kida [J. Number Theory **12** (1980), no. 4, 519–528; MR0599821 (82c:12006)], K. Iwasawa [Tôhoku Math. J. (2) **33** (1981), no. 2, 263–288; MR0624610 (83b:12003)], W. M. Sinnott [Compositio Math. **53** (1984), no. 1, 3–17; MR0762305 (86e:11103)], K. Wingberg [Comment. Math. Helv. **63** (1988), no. 4, 587–592; MR0966951 (90b:11061)], Y. Hachimori and K. Matsuno [J. Algebraic Geom. **8** (1999), no. 3, 581–601; MR1689359 (2000c:11086)], and Matsuno [J. Number Theory **84** (2000), no. 1, 80–92; MR1782263 (2001g:11085)].

The theorem is reduced to the case where F'_∞/F_∞ is abelian. Then $\mathrm{Sel}(F'_\infty, A)$ is approximated by the sum of the Selmer groups $\mathrm{Sel}(F_\infty, A_\chi)$ of the twisted Galois modules A_χ by the characters χ of $G = \mathrm{Gal}(F'_\infty/F_\infty)$. These $\mathrm{Sel}(F_\infty, A_\chi)$ are "congruent" to each other if G has p-power order. The theorem is then proved by using the formulas of Weston [Manuscripta Math. **118** (2005), no. 2, 161–180; MR2177683 (2006k:11211)] and of M. Emerton, Pollack and Weston [Invent. Math. **163** (2006), no. 3, 523–580; MR2207234 (2007a:11059)] relating the λ-invariants of congruent Galois representations.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.