Kida’s formula and congruences. (English summary)


A Kida-type formula (an analogue for Iwasawa \( \lambda \)-invariants of the Riemann-Hurwitz genus formula) is proved for the Selmer groups of a rather general class of \( p \)-adic representations. Let \( F \) be a number field which is totally real or totally imaginary, \( p \) an odd prime number, \( V \) a nearly ordinary \( p \)-adic Galois representation of \( F \) (i.e. of \( G_F = \text{Gal}(\overline{F}/F) \)) defined over a sufficiently large finite extension \( K \) of \( \mathbb{Q}_p \), and \( T \) a \( G_F \)-stable \( \mathfrak{O} \)-lattice in \( V \), where \( \mathfrak{O} \) is the ring of integers of \( K \). Set \( A = V/T \). Then one can define the Selmer group \( \text{Sel}(F_\infty, A) \) of \( A \) over the cyclotomic \( \mathbb{Z}_p \)-extension \( F_\infty \) of \( F \). It is a module over \( \Lambda = \mathfrak{O}[[\text{Gal}(F_\infty/F)]] \), and its algebraic Iwasawa invariants \( \lambda(F_\infty, A) \) and \( \mu(F_\infty, A) \) are defined. If \( F'/F \) is a finite extension and \( F'_\infty = F'F_\infty \), then \( \text{Sel}(F'_\infty, A) \) and its invariants \( \lambda(F'_\infty, A) \) and \( \mu(F'_\infty, A) \) are defined similarly. The main result is: Theorem. Let \( F'/F \) be a finite Galois extension of a \( p \)-power degree. Assume that \( T \) satisfies some technical assumptions. If \( \text{Sel}(F_\infty, A) \) is \( \Lambda \)-cotorsion with algebraic \( \mu \)-invariant zero, then so is \( \text{Sel}(F'_\infty, A) \). Moreover, in this case, one has \( \lambda(F'_\infty, A) = [F'_\infty:F_\infty] \cdot \lambda(F_\infty, A) + \sum_{w'} m_{w'}(V) \).

Here, the sum is over the places \( w' \) of \( F'_\infty \) which are prime to \( p \) and ramified in \( F'_\infty/F_\infty \), and \( m_{w'}(V) \) is a certain local invariant of \( V \) defined in terms of the behavior of \( V \) when twisted by characters of \( \text{Gal}(F'_\infty/F_\infty) \). If \( V \) is associated to a cuspidal (elliptic modular) eigenform \( f \) and \( F' \) is abelian over \( \mathbb{Q} \), then the same results hold for the analytic Iwasawa invariants of \( f \).


The theorem is reduced to the case where \( F'_\infty/F_\infty \) is abelian. Then \( \text{Sel}(F'_\infty, A) \) is approximated by the sum of the Selmer groups \( \text{Sel}(F'_\infty, A_\chi) \) of the twisted Galois modules \( A_\chi \) by the characters \( \chi \) of \( G = \text{Gal}(F'_\infty/F_\infty) \). These \( \text{Sel}(F'_\infty, A_\chi) \) are “congruent” to each other if \( G \) has \( p \)-power order. The theorem is then proved by using the formulas of Weston [Manuscripta Math. 118 (2005), no. 2, 161–180; MR2177683 (2006k:11211)] and of M. Emerton, Pollack and Weston [Invent. Math. 163 (2006), no. 3, 523–580; MR2207234 (2007a:11059)] relating the \( \lambda \)-invariants of congruent Galois representations.

Reviewed by Yuichiro Taguchi

References

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

© Copyright American Mathematical Society 2008