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Variation of Iwasawa invariants in Hida families.

The aim of this important paper is to study the variation of Iwasawa invariants of modular forms in Hida families.

Let \( f \) be a modular form and let \( p \) be a prime at which \( f \) is ordinary and \( p \)-distinguished. One can associate to \( f \) two kinds of \( p \)-adic power series. The first, \( L^\text{an}_f \), is the \( p \)-adic \( L \)-function, which is designed to interpolate the special values of the usual \( L \)-function associated to \( f \). The second, \( L^\text{alg}_f \), is the characteristic series of the dual of the Selmer group of \( f \). It is a generalization of a conjecture of B. Mazur [cf. Invent. Math. 18 (1972), 183–266; MR0444670 (56 #3020)] that \( L^\text{alg}_f \) and \( L^\text{an}_f \) are equal up to a unit: this is the “main conjecture of Iwasawa theory”. K. Katô has proved in many cases [cf. Astérisque No. 295 (2004), ix, 117–290; MR2104361 (2006b:11051)] that \( L^\text{alg}_f \) divides \( L^\text{an}_f \) up to a power of \( p \). For \( * \in \{ \text{alg, an} \} \), let \( \mu^*(f) \) be the exponent of the highest power of \( p \) dividing \( L^*_f \) and let \( \lambda^*(f) \) be the number of zeroes of \( L^*_f \). Katô’s result implies that
\[
\mu^\text{alg}(f) \leq \mu^\text{an}(f) \quad \text{and} \quad \lambda^\text{alg}(f) \leq \lambda^\text{an}(f)
\]
and the main conjecture is equivalent to equality in both cases. It therefore represents important progress towards the main conjecture to be able to prove something about those invariants, and this is what the authors do in this paper.

Let \( \overline{\rho} \) be the mod \( p \) representation attached to \( f \) and let \( \mathcal{H}(\overline{\rho}) \) be the Hida family of \( \overline{\rho} \), that is, the set of all \( p \)-ordinary \( p \)-stabilized newforms \( f \) with mod \( p \) Galois representation isomorphic to \( \overline{\rho} \). These newforms are dense in a \( p \)-adic analytic space of overconvergent eigenforms, consisting of an intersecting system of branches \( T(\alpha) \) indexed by the minimal primes \( \alpha \) of a certain Hecke algebra. The main results of the paper concern the variation of the invariants \( \mu^*(f) \) and \( \lambda^*(f) \) as \( f \) varies in \( \mathcal{H}(\overline{\rho}) \). For \( * \in \{ \text{alg, an} \} \), the two main theorems are:

Theorem 1. If \( \mu^*(f_0) = 0 \) for some \( f_0 \in \mathcal{H}(\overline{\rho}) \), then \( \mu^*(f) = 0 \) for all \( f \in \mathcal{H}(\overline{\rho}) \) (we then write \( \mu^*(\overline{\rho}) = 0 \)).

Theorem 2. If \( \mu^*(\overline{\rho}) = 0 \), and if \( f_1 \) and \( f_2 \) lie on two branches \( T(\alpha_1) \) and \( T(\alpha_2) \), then \( \lambda^*(f_1) - \lambda^*(f_2) = \sum_{l|N_1N_2} e_l(\alpha_2) - e_l(\alpha_1) \) where \( e_l(\alpha) \) is some explicit invariant of \( T(\alpha) \).

We refer to the article’s informative introduction for an explanation of the proofs and for some corollaries of Theorems 1 and 2 above. The last section (§5.3) of the article consists of numerical examples and some open questions.

Reviewed by Laurent N. Berger

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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