Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something completely different.
—Goethe

Honors Calculus – Math 129 – Fall 2009 – R. Pollack
HW #5

1. Compute the following limits. Prove that your answers are correct directly from the $\varepsilon - \delta$ definition of limits.
   (a) $\lim_{x \to 2} x^3$
   (b) $\lim_{x \to 1} \sqrt{x}$
   (c) $\lim_{x \to 3} \frac{1}{x^2}$
2. Give an example of functions $f(x)$ and $g(x)$ such that neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ exist, but $\lim_{x \to a} f(x) + g(x)$ exists.
3. Consider the function
$$f(x) = \begin{cases} x & \text{is rational,} \\ 0 & \text{is irrational.} \end{cases}$$
   Prove that $\lim_{x \to 0} f(x) = 0$. Does $\lim_{x \to a} f(x)$ exist for any other values of a?
4. Consider the following modified definition of a limit: We say a function $f(x)$ *quasi-approaches* L as x approaches a if for all $\delta > 0$, there exists an $\varepsilon > 0$, such that for all $x \in \mathbb{R}$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$. [Note that the ε and δ are switched!]
   (a) Give an example of a function $f(x)$ and real numbers a and L such that $f(x)$ quasi-approaches L as x approaches a, but for which it is not true that $\lim_{x \to a} f(x) = L$.
   (b) Give an example of a function $f(x)$ and real numbers a and L such that $\lim_{x \to a} f(x) = L$, but for which it is not true that $f(x)$ quasi-approaches L as x approaches a.
5. Spivak, Chapter 5: 4,7
6. (Challenge problem) Spivak, Chapter 5: 5(a)