Introduction to Analysis – MA 511 – Fall 2018 – R. Pollack
HW #1

1. Prove that $\mathbb{Q}$ is closed under addition and multiplication. That is, show that if $x, y \in \mathbb{Q}$, then $x+y \in \mathbb{Q}$ and $xy \in \mathbb{Q}$.

2. What error occurs in the below "proof" to cause such a ridiculous conclusion? Let $x = y$. Then

$$
\begin{aligned}
x^2 &= xy \\
x^2 - y^2 &= xy - y^2 \\
(x+y)(x-y) &= y(x-y) \\
x + y &= y \\
2y &= y \\
2 &= 1
\end{aligned}
$$

3. Let $\mathbb{I}$ denote the set of irrational numbers (that is the set of real numbers which are not in $\mathbb{Q}$).

   (a) Is $\mathbb{I}$ closed under addition? If so, prove it. If not, give a counter-example.

   (b) Is $\mathbb{I}$ closed under multiplication? If so, prove it. If not, give a counter-example.

   (c) Rudin: Chapter 1, exercise 1.

4. Consider the following relation on $\mathbb{R}^2$. We say

$$(x, y) < (x', y') \text{ if } x < x' \text{ or if both } x = x' \text{ and } y < y'.$$

   Under this relation, which of the following are true? Explain.

   (a) $(1, 2) < (2, 3)$

   (b) $(1, 2) < (3, 1)$

   (c) $(1, 2) < (1, 1)$

   (d) $(1, 2) < (1, 3)$

   Does this relation give an order on $\mathbb{R}^2$ (in the sense of Rudin, definition 1.5)?