Introduction to Analysis – MA 511 – Fall 2018 – R. Pollack
HW #4 (expanded)

Questions from Rudin:
Chapter 2 – 1,4,9,10,11
(Hint: for 4 see Theorem 2.12.)

Additional questions:

1. For $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in $\mathbb{R}^2$, define
$$d(x, y) = |x_1 - y_1| + |x_2 - y_2|.$$
Prove this function (the "taxi-cab" metric) is indeed a metric.

2. Recall that if E is a subset of a metric space, then $\overline{E}$ denotes the closure of E and E° denotes the interior of E. If $E = \mathbb{Q}$ thought of as a subset of $\mathbb{R}$ with the standard metric, determine $\overline{E}$ and E°. Justify your answers.

3. Let A and B be subsets of a metric space. Prove that
$$\overline{A \cup B} = \overline{A} \cup \overline{B},$$
that is, the closure of $A \cup B$ equals the closure of A union the closure of B.
(Hint: Theorem 2.27c will be helpful here.)

4. Let A_1, A_2, ... be subsets of a metric space. Is it true that
$$\overline{\bigcup_{i=1}^{\infty} A_i} = \bigcup_{i=1}^{\infty} \overline{A_i}?$$
If so, prove it. If not, give a counter-example, and point out what part of your proof of #3 breaks down.