Introduction to Analysis – MA 511 – Fall 2018 – R. Pollack
HW #7

Questions from Rudin: Chapter 3: 6(a,b), 9, 10, 23, 24(a,b)

Additional questions:

1. Let $f : X \to Y$ be a function between two sets. For a subset $A \subseteq X$, we define

$$f(A) = \{b \in B \text{ such that } b = f(a) \text{ for some } a \in A\},$$

and for a subset $B \subseteq Y$, we define

$$f^{-1}(B) = \{a \in A \text{ such that } f(a) \in B\}.$$

In each of the following pairs of sets, the two sets are related by either $=$, $\subset$ or $\supset$. Determine which is the correct relation and prove your answer. If you answer $\subset$ or $\supset$, give an explicit example where the reverse inclusion does not hold.

   (a) $f(A_1 \cup A_2) \longleftrightarrow f(A_1) \cup f(A_2)$
   (b) $f(A_1 \cap A_2) \longleftrightarrow f(A_1) \cap f(A_2)$
   (c) $f^{-1}(B_1 \cup B_2) \longleftrightarrow f^{-1}(B_1) \cup f^{-1}(B_2)$
   (d) $f^{-1}(B_1 \cap B_2) \longleftrightarrow f^{-1}(B_1) \cap f^{-1}(B_2)$
   (e) $f(f^{-1}(B)) \longleftrightarrow B$
   (f) $f^{-1}(f(A)) \longleftrightarrow A$

2. Let $\{a_n\}$ be a convergent sequence in $\mathbb{R}$. Prove that

$$\limsup\{a_n\} = \liminf\{a_n\} = \lim\{a_n\}.$$

3. **(Optional challenge problems)** Rudin: Chapter 3: 24(c,d,e)