

Each problem that I solved became a rule which served afterwards to solve other problems.
—Descartes

Modern Algebra – Math 541 – Fall 2009 – R. Pollack
HW #4

1. Use the Euclidean algorithm to compute the greatest common divisor of 320353 and 257642.
2. Prove that the equation

$$320353x + 257642y = 1$$

has no solutions with x, y in \mathbb{Z} . (Please do this question directly without referring to theorems from class.)

3. Compute the following values: $\phi(100)$, $\phi(40)$, $\phi(101)$.
[Recall that we have formulas $\phi(p^n) = p^n - p^{n-1}$ and $\phi(mn) = \phi(m)\phi(n)$ if m and n are relatively prime.]
4. Give 5 examples of groups with 8 elements. Do these groups have “distinct” multiplication tables up to reordering?
5. Compute the order of $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ in $\text{GL}_2(\mathbb{Z}_3)$ – that is, find the smallest positive integer d such that A^d is the identity matrix.
6. The group $U(17)$ has 16 elements. Thus, for any element $a \in U(17)$, we have that the order of a divides 16 (as proven in class). Is the converse true? That is, for each positive divisor d of 16, try to find an element of order d . Try this question also for $U(15)$.
7. Let a and b be elements of $U(m)$. Let e be the order of a and let f be the order of b . Prove that the order of ab divides ef . Give an example where the order of ab is smaller than ef .