By indirections find directions out. – Shakespeare

Modern Algebra – Math 541 – Fall 2009 – R. Pollack
HW #7

1. Let G be a group and let a be an element of G with order n.
   (a) Prove $(bab^{-1})^n = e$.
   (b) Prove that the order of bab^{-1} is exactly n.
2. Let G be a group such that $a^2 = e$ for all $a \in G$. Prove that G is abelian.
   [Hint: To show $ab = ba$ use the fact that $a^2 = b^2 = (ab)^2 = e$.]
3. Let G be a group of size 6 with 5 elements of order 2 and 0 elements of order 3 – *i.e.* case 2a.
   (a) Prove that G is abelian.
   (b) Let a and b be two distinct elements in G with order 2. Prove that ab does not equal e, a or b.
   (c) Prove that the 4 element collection $\{e, a, b, ab\}$ is a subgroup of G.
   (d) Deduce that no such group G exists (completing the proof from class).
4. For each of the following groups of size 8
   $$\mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, D_4, Q_8$$
   determine the number of elements of each possible order from 1 to 8.
5. For $a, b \in G$, we say a is *conjugate* to b if there exists some $x \in G$ such that $a = xbx^{-1}$.
   (a) Prove that if a is conjugate to b, then b is conjugate to a.
   (b) Prove that if a is conjugate to b, then there exists some $y \in G$ such that $a = y^{-1}by$.
   (c) Prove that a is conjugate to a.
   (d) Prove that if a is conjugate to b and b is conjugate to c, then a is conjugate to c.
   (e) Determine all g in S_3 which are conjugate to (12).
   (f) Determine all g in S_3 which are conjugate to (123).
   (g) Determine all g in D_4 which are conjugate to R_{90}.
6. Gallian Chapter 7: 1,2,26,28,38 (7th Edition)
   Gallian Chapter 7: 1,2,24,26,34 (6th Edition)