Topology – MA 564 – Spring 2015 – R. Pollack HW #7

Complete each of the following exercises.

- 1. Prove that the interval (a, b) is path connected.
- 2. Prove that $\mathbb{R} \{0\}$ is not path connected.
- 3. Prove that $\mathbb{R}^2 \mathbb{Q}^2$ is path connected.
- 4. Prove that if $f: X \to Y$ is continuous and X is path connected, then f(X) is path connected.
- 5. Prove that the unit circle S^1 is path connected.
- 6. Let G_1 and G_2 be groups and let $\varphi : G_1 \to G_2$ be an isomorphism (i.e. a bijective homomorphism). Prove that φ^{-1} is also a homomorphism.
- 7. As in class, for $x \in X$, let γ_x denote the constant path at x given by $\gamma_x(t) = x$ for all $t \in [0, 1]$. Let γ denote any path from a to b. Check that $\gamma_a \star \gamma \sim \gamma$ and $\gamma \star \gamma_b \sim \gamma$.

Gamelin & Greene, Chapter 3, section 3: 3 and 5