

Algebraic Number Theory
MA844 (aka MA743)
Spring 2014
HW #2

- (1) Show that $\mathbb{Z}[\sqrt{2}]$ is dense in $\mathbb{R}$.
- (2) Let K be a number field with α in $\mathcal{O}_K$. Prove that if $N(\alpha) = \pm 1$ then α is a unit in $\mathcal{O}_K$.
- (3) Let K be a number field and let $\alpha \in K$.
 - (a) Find an example of a K and an α such that there is an embedding of $\sigma : K \hookrightarrow \mathbb{C}$ such that $|\sigma(\alpha)| = 1$ but α is not a root of unit.
 - (b) Prove that if $|\sigma(\alpha)| = 1$ for all embeddings $\sigma : K \rightarrow \mathbb{C}$ then α is a root of unity. (For hints see Marcus Chapter 2: 11)
- (4) Marcus Chapter 1: 16–29 (exercises related to Fermat’s last theorem)
- (5) Marcus Chapter 2: 2, 15, 29
- (6) Neukrich Chapter 1, section 2: 1,3
- (7) Neukrich Chapter 1, section 3: 1,2,3,9