

# ACCURACY OF GHOST SERIES ( $p = 3$ AND $N = 1$ )

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ABSTRACT. We present data showing how “accurate” the ghost series predictions are.

We present a series of tables on the first 12 coefficients of the ghost series for  $p = 3$  and level  $N = 1$ . The tables take the form:

TABLE 0.1. Sample (separated) table

$k$	$m_i(k)$	relative loc. of zeros
$\vdots$	$\vdots$	$\vdots$
20	0	6, 6, 5, 5, 4, ...
22	0	6, 6, 5, 5, 4, ...
24	1	<b><u>9</u></b> 6, 5, 5, 4, ...
26	1	<b><u>9</u></b> 6, 5, 5, 4, ...
28	2	<b><u>10</u></b> <b><u>7</u></b> 5, 5, 4, ...
30	3	<b><u>11</u></b> <b><u>10</u></b> <b><u>8</u></b> 5, 4, ...
32	2	<b><u>16</u></b> <b><u>13</u></b> 6, 5, 4, ...
34	0	6, 6, 5, 5, 4, ...
36	1	<b><u>14</u></b> 6, 5, 5, 4, ...
38	0	6, 6, 5, 5, 4, ...
40	0	6, 6, 5, 5, 4, ...
$\vdots$	$\vdots$	$\vdots$

The first column is a list of (even) integers  $k$ . The second column is the multiplicity of  $k$  as a zero of the ghost series in the  $i$ -th index. The third column is the (decreasing) list of numbers  $v_p(w_\kappa - w_k)$  where  $\kappa$  runs over the finitely many solutions to  $\text{tr}(\wedge^i U_p)(\kappa) = 0$ . For a given  $k$ , if  $m_i(k) > 0$  then we have bolded, underlined and separated out the largest  $m_i(k)$ -many values in the third column to illustrate the link between the “ghost zeros” and the true zeros of the characteristic series of  $U_p$ .

The data is truncated in the following two ways. First, list of  $k$  are exactly those within 10 of some predicted zero of the ghost coefficient. Second, the number of terms in the third column is always exactly two more than the highest multiplicity of a ghost zero.

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## 1. THE TABLES

TABLE 1.1. Coefficient  $i = 1$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros
0	0	1, 1, 0, ...
2	0	2, 1, 0, ...
4	0	2, 1, 0, ...
6	0	1, 1, 0, ...
8	0	2, 1, 0, ...
10	1	<u>8</u> 1, 0, ...
12	0	1, 1, 0, ...
14	1	<u>5</u> 1, 0, ...
16	0	2, 1, 0, ...
18	0	1, 1, 0, ...
20	0	2, 1, 0, ...
22	0	2, 1, 0, ...
24	0	1, 1, 0, ...

TABLE 1.2. Coefficient  $i = 2$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros
4	0	3, 2, 1, ...
6	0	2, 1, 1, ...
8	0	3, 2, 2, ...
10	0	2, 2, 1, ...
12	0	2, 1, 1, ...
14	1	<u>5</u> 2, 2, ...
16	1	<u>8</u> 2, 1, ...
18	1	<u>5</u> 1, 1, ...
20	1	<u>5</u> 2, 2, ...
22	1	<u>14</u> 2, 1, ...
24	0	2, 1, 1, ...
26	1	<u>8</u> 2, 2, ...
28	0	2, 2, 1, ...
30	0	2, 1, 1, ...
32	0	3, 2, 2, ...
34	0	3, 2, 1, ...
36	0	3, 1, 1, ...

TABLE 1.3. Coefficient  $i = 3$  for  $p = 3$  and tame level  $N = 1$

$k$	pred. mult.	rel. pos. true zeros	
8	0		3, 3, 2, 2, ...
10	0		3, 2, 2, 2, ...
12	0		3, 2, 2, 1, ...
14	0		3, 2, 2, 2, ...
16	0		3, 2, 2, 2, ...
18	1		<u>5</u> 2, 2, 1, ...
20	1		<u>5</u> 3, 2, 2, ...
22	2	<u>5.5</u>	<u>5.5</u> 2, 2, ...
24	1		<u>5</u> 2, 2, 1, ...
26	2	<u>8</u>	<u>6</u> 2, 2, ...
28	1		<u>16</u> 2, 2, 2, ...
30	1		<u>8</u> 2, 2, 1, ...
32	1		<u>10</u> 2, 2, 2, ...
34	1		<u>28</u> 2, 2, 2, ...
36	0		3, 2, 2, 1, ...
38	1		<u>12</u> 3, 2, 2, ...
40	0		3, 3, 2, 2, ...
42	0		3, 2, 2, 1, ...
44	0		3, 3, 2, 2, ...
46	0		3, 2, 2, 2, ...
48	0		3, 2, 2, 1, ...

TABLE 1.4. Coefficient  $i = 4$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros	
12	0		3, 3, 2, 2, ...
14	0		3, 3, 3, 2, ...
16	0		3, 3, 2, 2, ...
18	0		3, 2, 2, 2, ...
20	0		3, 3, 2, 2, ...
22	1	<u>14</u>	3, 2, 2, ...
24	1	<u>5</u>	3, 2, 2, ...
26	2	<u>8</u> <u>6</u>	3, 2, ...
28	2	<u>5</u> <u>5</u>	3, 2, ...
30	2	<u>8</u> <u>6</u>	2, 2, ...
32	2	<u>10</u> <u>5</u>	3, 2, ...
34	2	<u>9.5</u> <u>9.5</u>	2, 2, ...
36	1		<u>10</u> 2, 2, 2, ...
38	2	<u>13</u> <u>10</u>	2, 2, ...
40	1		<u>31</u> 3, 2, 2, ...
42	1		<u>12</u> 3, 2, 2, ...
44	1		<u>12</u> 3, 3, 2, ...
46	1		<u>25</u> 3, 3, 2, ...
48	0		3, 3, 2, 2, ...
50	1	<u>14</u>	3, 3, 2, ...
52	0		3, 3, 2, 2, ...
54	0		3, 2, 2, 2, ...
56	0		3, 3, 2, 2, ...
58	0		3, 3, 2, 2, ...
60	0		3, 3, 2, 2, ...

TABLE 1.5. Coefficient  $i = 5$  for  $p = 3$  and tame level  $N = 1$

$k$	pred. mult.	rel. pos.		true zeros
16	0			3, 3, 3, 3, 2, ...
18	0			3, 3, 3, 2, 2, ...
20	0			3, 3, 3, 3, 2, ...
22	0			3, 3, 3, 2, 2, ...
24	0			3, 3, 2, 2, 2, ...
26	1		<u>8</u>	3, 3, 3, 2, ...
28	1		<u>16</u>	3, 3, 2, 2, ...
30	2		<u>8</u> <u>6</u>	3, 2, 2, ...
32	2		<u>10</u> <u>5</u>	3, 3, 2, ...
34	3	<u>22</u>	<u>5.5</u> <u>5.5</u>	3, 2, ...
36	2		<u>10</u> <u>5</u>	3, 2, 2, ...
38	3	<u>13</u>	<u>10</u> <u>5</u>	3, 2, ...
40	2		<u>10</u> <u>10</u>	3, 2, 2, ...
42	2		<u>13</u> <u>10</u>	2, 2, 2, ...
44	2		<u>13</u> <u>9</u>	3, 3, 2, ...
46	2		<u>12</u> <u>12</u>	3, 2, 2, ...
48	1		<u>12</u>	3, 3, 2, 2, ...
50	2		<u>14</u> <u>12</u>	3, 3, 2, ...
52	1		<u>26</u>	3, 3, 3, 2, ...
54	1		<u>14</u>	3, 3, 2, 2, ...
56	1		<u>14</u>	3, 3, 3, 2, ...
58	1		<u>34</u>	3, 3, 2, 2, ...
60	0			3, 3, 2, 2, 2, ...
62	1		<u>18</u>	3, 3, 3, 2, ...
64	0			3, 3, 3, 2, 2, ...
66	0			3, 3, 3, 2, 2, ...
68	0			3, 3, 3, 3, 2, ...
70	0			3, 3, 3, 3, 2, ...
72	0			3, 3, 3, 2, 2, ...

TABLE 1.6. Coefficient  $i = 6$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros		
20	0			4, 3, 3, 3, 3, ...
22	0			3, 3, 3, 3, 3, ...
24	0			3, 3, 3, 3, 2, ...
26	0			3, 3, 3, 3, 3, ...
28	0			3, 3, 3, 3, 2, ...
30	1		<u>8</u>	3, 3, 3, 2, ...
32	1		<u>10</u>	3, 3, 3, 3, ...
34	2	<u>9.5</u>	<u>9.5</u>	3, 3, 3, ...
36	2	<u>10</u>	<u>5</u>	3, 3, 2, ...
38	3	<u>13</u>	<u>10</u>	<u>5</u> 3, 3, ...
40	3	<u>22</u>	<u>6</u>	<u>6</u> 3, 3, ...
42	3	<u>13</u>	<u>10</u>	<u>5</u> 3, 2, ...
44	3	<u>13</u>	<u>9</u>	<u>7</u> 3, 3, ...
46	3	<u>25</u>	<u>10.5</u>	<u>10.5</u> 3, 2, ...
48	2		<u>13</u>	<u>9</u> 3, 3, 2, ...
50	3	<u>13</u>	<u>13</u>	<u>10</u> 3, 3, ...
52	2		<u>12</u>	<u>12</u> 3, 3, 3, ...
54	2		<u>14</u>	<u>12</u> 3, 3, 2, ...
56	2		<u>14</u>	<u>12</u> 3, 3, 3, ...
58	2		<u>15</u>	<u>15</u> 3, 3, 3, ...
60	1			<u>14</u> 3, 3, 3, 2, ...
62	2		<u>18</u>	<u>16</u> 3, 3, 3, ...
64	1			<u>35</u> 3, 3, 3, 2, ...
66	1			<u>18</u> 3, 3, 3, 2, ...
68	1			<u>19</u> 3, 3, 3, 3, ...
70	1			<u>40</u> 3, 3, 3, 3, ...
72	0			3, 3, 3, 3, 2, ...
74	1		<u>21</u>	3, 3, 3, 3, ...
76	0			3, 3, 3, 3, 3, ...
78	0			3, 3, 3, 3, 2, ...
80	0			3, 3, 3, 3, 3, ...
82	0			3, 3, 3, 3, 2, ...
84	0			4, 3, 3, 3, 2, ...

TABLE 1.7. Coefficient  $i = 7$  for  $p = 3$  and tame level  $N = 1$

$k$	pred. mult.	rel. pos. true zeros			
24	0				4, 3, 3, 3, 3, 3, ...
26	0				4, 3, 3, 3, 3, 3, ...
28	0				4, 3, 3, 3, 3, 3, ...
30	0				3, 3, 3, 3, 3, 2, ...
32	0				4, 3, 3, 3, 3, 3, ...
34	1			<u>21</u>	3, 3, 3, 3, 3, ...
36	1			<u>10</u>	3, 3, 3, 3, 2, ...
38	2			<u>13</u> <u>10</u>	3, 3, 3, 3, ...
40	2			<u>10</u> <u>10</u>	3, 3, 3, 3, ...
42	3		<u>13</u>	<u>10</u> <u>5</u>	3, 3, 3, ...
44	3		<u>13</u>	<u>9</u> <u>7</u>	3, 3, 3, ...
46	4	<u>12.5</u>	<u>12.5</u>	<u>5.5</u> <u>5.5</u>	3, 3, ...
48	3		<u>13</u>	<u>9</u> <u>7</u>	3, 3, 2, ...
50	4	<u>14</u>	<u>13</u>	<u>10</u> <u>6</u>	3, 3, ...
52	3		<u>26</u>	<u>10</u> <u>10</u>	3, 3, 3, ...
54	3		<u>13</u>	<u>13</u> <u>10</u>	3, 3, 2, ...
56	3		<u>13</u>	<u>13</u> <u>10</u>	3, 3, 3, ...
58	3		<u>34</u>	<u>12</u> <u>12</u>	3, 3, 3, ...
60	2			<u>14</u> <u>12</u>	3, 3, 3, 3, ...
62	3		<u>18</u>	<u>16</u> <u>12</u>	3, 3, 3, ...
64	2			<u>14.5</u> <u>14.5</u>	3, 3, 3, 3, ...
66	2			<u>18</u> <u>16</u>	3, 3, 3, 2, ...
68	2			<u>19</u> <u>15</u>	3, 3, 3, 3, ...
70	2			<u>19</u> <u>19</u>	3, 3, 3, 3, ...
72	1			<u>19</u>	3, 3, 3, 3, 2, ...
74	2			<u>22</u> <u>19</u>	3, 3, 3, 3, ...
76	1			<u>40</u>	3, 3, 3, 3, 3, ...
78	1			<u>21</u>	3, 3, 3, 3, 3, ...
80	1			<u>21</u>	3, 3, 3, 3, 3, ...
82	1			<u>45</u>	3, 3, 3, 3, 3, ...
84	0				3, 3, 3, 3, 3, 2, ...
86	1			<u>25</u>	3, 3, 3, 3, 3, ...
88	0				4, 3, 3, 3, 3, 3, ...
90	0				4, 3, 3, 3, 3, 2, ...
92	0				4, 4, 3, 3, 3, 3, ...
94	0				4, 4, 3, 3, 3, 3, ...
96	0				4, 4, 4, 3, 3, 3, ...

TABLE 1.8. Coefficient  $i = 8$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros				
28	0				4, 4, 3, 3, 3, 3, ...	
30	0				4, 3, 3, 3, 3, 3, ...	
32	0				4, 4, 3, 3, 3, 3, ...	
34	0				4, 3, 3, 3, 3, 3, ...	
36	0				4, 3, 3, 3, 3, 3, ...	
38	1			<u>12</u>	4, 3, 3, 3, 3, ...	
40	1			<u>21</u>	4, 3, 3, 3, 3, ...	
42	2		<u>13</u>	<u>10</u>	3, 3, 3, 3, ...	
44	2		<u>13</u>	<u>9</u>	4, 3, 3, 3, ...	
46	3	<u>24</u>	<u>10.5</u>	<u>10.5</u>	3, 3, 3, ...	
48	3	<u>13</u>	<u>9</u>	<u>7</u>	3, 3, 3, ...	
50	4	<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	3, 3, ...
52	4	<u>12.5</u>	<u>12.5</u>	<u>6.5</u>	<u>6.5</u>	3, 3, ...
54	4	<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	3, 3, ...
56	4	<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	3, 3, ...
58	4	<u>14.5</u>	<u>14.5</u>	<u>10</u>	<u>10</u>	3, 3, ...
60	3		<u>13</u>	<u>13</u>	<u>10</u>	3, 3, 3, ...
62	4	<u>18</u>	<u>15</u>	<u>13</u>	<u>9</u>	3, 3, ...
64	3		<u>35</u>	<u>12</u>	<u>12</u>	3, 3, 3, ...
66	3		<u>18</u>	<u>16</u>	<u>12</u>	3, 3, 3, ...
68	3		<u>19</u>	<u>15</u>	<u>12</u>	3, 3, 3, ...
70	3		<u>40</u>	<u>15.5</u>	<u>15.5</u>	3, 3, 3, ...
72	2			<u>19</u>	<u>15</u>	3, 3, 3, 3, ...
74	3		<u>22</u>	<u>19</u>	<u>15</u>	3, 3, 3, ...
76	2			<u>19</u>	<u>19</u>	3, 3, 3, 3, ...
78	2			<u>22</u>	<u>19</u>	3, 3, 3, 3, ...
80	2			<u>22</u>	<u>18</u>	3, 3, 3, 3, ...
82	2			<u>21</u>	<u>21</u>	3, 3, 3, 3, ...
84	1				<u>21</u>	3, 3, 3, 3, 3, ...
86	2			<u>25</u>	<u>21</u>	3, 3, 3, 3, ...
88	1				<u>48</u>	3, 3, 3, 3, 3, ...
90	1				<u>25</u>	3, 3, 3, 3, 3, ...
92	1				<u>25</u>	4, 3, 3, 3, 3, ...
94	1				<u>54</u>	4, 3, 3, 3, 3, ...
96	0					4, 4, 3, 3, 3, 3, ...
98	1				<u>28</u>	4, 4, 3, 3, 3, ...
100	0					4, 4, 4, 3, 3, 3, ...
102	0					4, 4, 4, 3, 3, 3, ...
104	0					4, 4, 4, 4, 3, 3, ...
106	0					4, 4, 4, 4, 3, 3, ...
108	0					4, 4, 4, 4, 3, 3, ...



TABLE 1.9. Coefficient  $i = 9$  for  $p = 3$  and tame level  $N = 1$

$k$	pred. mult.	rel. pos. true zeros					
32	0					4, 4, 4, 3, 3, 3, 3, ...	
34	0					4, 4, 3, 3, 3, 3, 3, ...	
36	0					4, 4, 3, 3, 3, 3, 3, ...	
38	0					4, 4, 3, 3, 3, 3, 3, ...	
40	0					4, 4, 3, 3, 3, 3, 3, ...	
42	1				<u>12</u>	4, 3, 3, 3, 3, 3, ...	
44	1				<u>12</u>	4, 4, 3, 3, 3, 3, ...	
46	2			<u>12</u>	<u>12</u>	4, 3, 3, 3, 3, ...	
48	2			<u>13</u>	<u>9</u>	4, 3, 3, 3, 3, ...	
50	3		<u>13</u>	<u>13</u>	<u>10</u>	4, 3, 3, 3, ...	
52	3		<u>25</u>	<u>10</u>	<u>10</u>	4, 3, 3, 3, ...	
54	4	<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	3, 3, 3, ...	
56	4	<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	4, 3, 3, ...	
58	5	<b>33</b>	<u>12.5</u>	<u>12.5</u>	<u>6.5</u>	<u>6.5</u>	3, 3, ...
60	4		<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	3, 3, 3, ...
62	5	<b>17</b>	<u>16</u>	<u>13</u>	<u>9</u>	<u>7</u>	3, 3, ...
64	4		<u>14</u>	<u>14</u>	<u>10.5</u>	<u>10.5</u>	3, 3, 3, ...
66	4		<u>18</u>	<u>15</u>	<u>13</u>	<u>9</u>	3, 3, 3, ...
68	4		<u>19</u>	<u>14</u>	<u>13</u>	<u>10</u>	3, 3, 3, ...
70	4		<u>19</u>	<u>19</u>	<u>12.5</u>	<u>12.5</u>	3, 3, 3, ...
72	3			<u>19</u>	<u>15</u>	<u>12</u>	3, 3, 3, 3, ...
74	4		<u>22</u>	<u>19</u>	<u>15</u>	<u>13</u>	3, 3, 3, ...
76	3			<u>40</u>	<u>15.5</u>	<u>15.5</u>	3, 3, 3, 3, ...
78	3			<u>22</u>	<u>19</u>	<u>15</u>	3, 3, 3, 3, ...
80	3			<u>22</u>	<u>18</u>	<u>16</u>	3, 3, 3, 3, ...
82	3			<u>61</u>	<u>19.5</u>	<u>19.5</u>	3, 3, 3, 3, ...
84	2				<u>22</u>	<u>18</u>	3, 3, 3, 3, 3, ...
86	3			<u>24</u>	<u>22</u>	<u>19</u>	3, 3, 3, 3, ...
88	2				<u>22</u>	<u>22</u>	3, 3, 3, 3, 3, ...
90	2				<u>25</u>	<u>21</u>	3, 3, 3, 3, 3, ...
92	2				<u>25</u>	<u>23</u>	3, 3, 3, 3, 3, ...
94	2				<u>25.5</u>	<u>25.5</u>	3, 3, 3, 3, 3, ...
96	1					<u>25</u>	4, 3, 3, 3, 3, 3, ...
98	2				<u>28</u>	<u>26</u>	4, 3, 3, 3, 3, ...
100	1					<u>55</u>	4, 4, 3, 3, 3, 3, ...
102	1					<u>28</u>	4, 4, 3, 3, 3, 3, ...
104	1					<u>29</u>	4, 4, 4, 3, 3, 3, ...
106	1					<u>59</u>	4, 4, 4, 3, 3, 3, ...
108	0						4, 4, 4, 4, 3, 3, 3, ...
110	1					<u>31</u>	4, 4, 4, 4, 3, 3, ...
112	0						4, 4, 4, 4, 4, 3, 3, ...
114	0						4, 4, 4, 4, 3, 3, 3, ...
116	0						4, 4, 4, 4, 4, 3, 3, ...
118	0						4, 4, 4, 4, 3, 3, 3, ...
120	0						4, 4, 4, 4, 3, 3, 3, ...

TABLE 1.10. Coefficient  $i = 10$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros				
36	0					4, 4, 4, 3, 3, 3, 3, ...
38	0					4, 4, 4, 3, 3, 3, 3, ...
40	0					4, 4, 4, 3, 3, 3, 3, ...
42	0					4, 4, 3, 3, 3, 3, 3, ...
44	0					4, 4, 4, 3, 3, 3, 3, ...
46	1				<u>25</u>	4, 4, 3, 3, 3, 3, ...
48	1				<u>12</u>	4, 4, 3, 3, 3, 3, ...
50	2			<u>14</u>	<u>12</u>	4, 4, 3, 3, 3, ...
52	2			<u>12</u>	<u>12</u>	4, 4, 3, 3, 3, ...
54	3			<u>13</u>	<u>13</u>	4, 3, 3, 3, ...
56	3			<u>13</u>	<u>13</u>	4, 4, 3, 3, ...
58	4	<u>14.5</u>	<u>14.5</u>	<u>10</u>	<u>10</u>	4, 3, 3, ...
60	4	<u>14</u>	<u>13</u>	<u>10</u>	<u>6</u>	4, 3, 3, ...
62	5	<u>17</u>	<u>16</u>	<u>13</u>	<u>9</u>	4, 3, ...
64	5	<u>34</u>	<u>12.5</u>	<u>12.5</u>	<u>5.5</u>	4, 3, ...
66	5	<u>17</u>	<u>16</u>	<u>13</u>	<u>9</u>	3, 3, ...
68	5	<u>18</u>	<u>15</u>	<u>13</u>	<u>10</u>	4, 3, ...
70	5	<u>39</u>	<u>15</u>	<u>15</u>	<u>10</u>	3, 3, ...
72	4		<u>19</u>	<u>14</u>	<u>13</u>	3, 3, 3, ...
74	5	<u>22</u>	<u>19</u>	<u>14</u>	<u>14</u>	3, 3, ...
76	4		<u>19</u>	<u>19</u>	<u>12.5</u>	3, 3, 3, ...
78	4		<u>22</u>	<u>19</u>	<u>15</u>	3, 3, 3, ...
80	4		<u>22</u>	<u>18</u>	<u>16</u>	3, 3, 3, ...
82	4		<u>21.5</u>	<u>21.5</u>	<u>15</u>	3, 3, 3, ...
84	3			<u>22</u>	<u>18</u>	3, 3, 3, 3, ...
86	4		<u>25</u>	<u>22</u>	<u>19</u>	3, 3, 3, ...
88	3			<u>67</u>	<u>19</u>	3, 3, 3, 3, ...
90	3			<u>24</u>	<u>22</u>	3, 3, 3, 3, ...
92	3			<u>24</u>	<u>24</u>	3, 3, 3, 3, ...
94	3			<u>54</u>	<u>22</u>	3, 3, 3, 3, ...
96	2				<u>25</u>	3, 3, 3, 3, 3, ...
98	3			<u>28</u>	<u>26</u>	3, 3, 3, 3, ...
100	2				<u>25</u>	4, 3, 3, 3, 3, ...
102	2				<u>28</u>	4, 3, 3, 3, 3, ...
104	2				<u>29</u>	4, 4, 3, 3, 3, ...
106	2			<u>29.5</u>	<u>29.5</u>	4, 4, 3, 3, 3, ...
108	1				<u>29</u>	4, 4, 4, 3, 3, 3, ...
110	2			<u>33</u>	<u>29</u>	4, 4, 4, 3, 3, ...
112	1				<u>59</u>	4, 4, 4, 4, 3, 3, ...
114	1				<u>31</u>	4, 4, 4, 4, 3, 3, ...
116	1				<u>31</u>	4, 4, 4, 4, 4, 3, ...
118	1				<u>61</u>	4, 4, 4, 4, 4, 3, ...
120	0					4, 4, 4, 4, 4, 3, 3, ...
122	1				<u>32</u>	4, 4, 4, 4, 4, 3, ...
124	0					4, 4, 4, 4, 4, 3, 3, ...
126	0					4, 4, 4, 4, 3, 3, 3, ...
128	0					4, 4, 4, 4, 4, 3, 3, ...
130	0					4, 4, 4, 4, 3, 3, 3, ...
132	0					4, 4, 4, 4, 3, 3, 3, ...

TABLE 1.11. Coefficient  $i = 11$  for  $p = 3$  and tame level  $N = 1$

$k$	pred. mult.	rel. pos. true zeros				
40	0					4, 4, 4, 4, 3, 3, 3, 3, ...
42	0					4, 4, 4, 3, 3, 3, 3, 3, ...
44	0					4, 4, 4, 4, 3, 3, 3, 3, ...
46	0					4, 4, 4, 3, 3, 3, 3, 3, ...
48	0					4, 4, 4, 3, 3, 3, 3, 3, ...
50	1				<u>14</u>	4, 4, 4, 3, 3, 3, 3, ...
52	1				<u>26</u>	4, 4, 4, 3, 3, 3, 3, ...
54	2			<u>14</u>	<u>12</u>	4, 4, 3, 3, 3, 3, ...
56	2			<u>14</u>	<u>12</u>	4, 4, 4, 3, 3, 3, ...
58	3		<u>34</u>	<u>12</u>	<u>12</u>	4, 4, 3, 3, 3, ...
60	3		<u>13</u>	<u>13</u>	<u>10</u>	4, 4, 3, 3, 3, ...
62	4		<u>18</u>	<u>15</u>	<u>13</u>	4, 4, 3, 3, ...
64	4		<u>14</u>	<u>14</u>	<u>10.5</u>	4, 4, 3, 3, ...
66	5	<u>17</u>	<u>16</u>	<u>13</u>	<u>9</u>	4, 3, 3, ...
68	5	<u>18</u>	<u>15</u>	<u>13</u>	<u>10</u>	4, 4, 3, ...
70	6	<u>18</u>	<u>13</u>	<u>13</u>	<u>6</u>	4, 3, ...
72	5	<u>18</u>	<u>15</u>	<u>13</u>	<u>10</u>	4, 3, 3, ...
74	6	<u>21</u>	<u>18</u>	<u>15</u>	<u>14</u>	4, 3, ...
76	5		<u>39</u>	<u>15</u>	<u>15</u>	4, 3, 3, ...
78	5		<u>22</u>	<u>19</u>	<u>14</u>	3, 3, 3, ...
80	5		<u>22</u>	<u>18</u>	<u>15</u>	4, 3, 3, ...
82	5		<u>44</u>	<u>19.5</u>	<u>19.5</u>	3, 3, 3, ...
84	4		<u>22</u>	<u>18</u>	<u>16</u>	3, 3, 3, 3, ...
86	5	<u>25</u>	<u>22</u>	<u>19</u>	<u>15</u>	3, 3, 3, ...
88	4		<u>22.5</u>	<u>22.5</u>	<u>15.5</u>	3, 3, 3, 3, ...
90	4		<u>25</u>	<u>22</u>	<u>19</u>	3, 3, 3, 3, ...
92	4		<u>25</u>	<u>24</u>	<u>19</u>	3, 3, 3, 3, ...
94	4		<u>25</u>	<u>25</u>	<u>19</u>	3, 3, 3, 3, ...
96	3			<u>24</u>	<u>24</u>	3, 3, 3, 3, 3, ...
98	4		<u>28</u>	<u>25</u>	<u>24</u>	3, 3, 3, 3, ...
100	3			<u>55</u>	<u>23</u>	3, 3, 3, 3, 3, ...
102	3			<u>28</u>	<u>26</u>	3, 3, 3, 3, 3, ...
104	3			<u>29</u>	<u>25</u>	4, 3, 3, 3, 3, ...
106	3			<u>60</u>	<u>25.5</u>	4, 3, 3, 3, 3, ...
108	2			<u>29</u>	<u>25</u>	4, 4, 3, 3, 3, 3, ...
110	3			<u>33</u>	<u>29</u>	4, 4, 3, 3, 3, ...
112	2			<u>29.5</u>	<u>29.5</u>	4, 4, 4, 3, 3, 3, ...
114	2			<u>33</u>	<u>29</u>	4, 4, 4, 3, 3, 3, ...
116	2			<u>33</u>	<u>28</u>	4, 4, 4, 4, 3, 3, ...
118	2			<u>31</u>	<u>31</u>	4, 4, 4, 4, 3, 3, ...
120	1				<u>31</u>	4, 4, 4, 4, 4, 3, 3, ...
122	2			<u>32</u>	<u>31</u>	4, 4, 4, 4, 4, 3, ...
124	1				<u>62</u>	4, 4, 4, 4, 4, 4, 3, ...
126	1				<u>32</u>	4, 4, 4, 4, 4, 3, 3, ...
128	1				<u>32</u>	4, 4, 4, 4, 4, 4, 3, ...
130	1				<u>68</u>	4, 4, 4, 4, 4, 3, 3, ...
132	0					4, 4, 4, 4, 4, 3, 3, 3, ...
134	1				<u>35</u>	4, 4, 4, 4, 4, 3, 3, ...
136	0					4, 4, 4, 4, 4, 3, 3, 3, ...
138	0					4, 4, 4, 4, 3, 3, 3, 3, ...
140	0					4, 4, 4, 4, 4, 3, 3, 3, ...
142	0					4, 4, 4, 4, 3, 3, 3, 3, ...
144	0					4, 4, 4, 4, 3, 3, 3, 3, ...

TABLE 1.12. Coefficient  $i = 12$  for  $p = 3$  and tame level  $N = 1$ 

$k$	pred. mult.	rel. pos. true zeros						
44	0						4, 4, 4, 4, 4, 3, 3, 3, ...	
46	0						4, 4, 4, 4, 3, 3, 3, 3, ...	
48	0						4, 4, 4, 4, 3, 3, 3, 3, ...	
50	0						4, 4, 4, 4, 3, 3, 3, 3, ...	
52	0						4, 4, 4, 4, 3, 3, 3, 3, ...	
54	1					<u>14</u>	4, 4, 4, 3, 3, 3, 3, ...	
56	1					<u>14</u>	4, 4, 4, 4, 3, 3, 3, ...	
58	2				<u>15</u>	<u>15</u>	4, 4, 4, 3, 3, 3, ...	
60	2				<u>14</u>	<u>12</u>	4, 4, 4, 3, 3, 3, ...	
62	3			<u>18</u>	<u>16</u>	<u>12</u>	4, 4, 4, 3, 3, ...	
64	3			<u>35</u>	<u>12</u>	<u>12</u>	4, 4, 4, 3, 3, ...	
66	4		<u>18</u>	<u>15</u>	<u>13</u>	<u>9</u>	4, 4, 3, 3, ...	
68	4		<u>19</u>	<u>14</u>	<u>13</u>	<u>10</u>	4, 4, 4, 3, ...	
70	5	<u>41</u>	<u>15</u>	<u>15</u>	<u>10</u>	<u>10</u>	4, 4, 3, ...	
72	5	<u>18</u>	<u>15</u>	<u>13</u>	<u>10</u>	<u>5</u>	4, 4, 3, ...	
74	6	<u>21</u>	<u>18</u>	<u>15</u>	<u>14</u>	<u>10</u>	<u>5</u>	4, 4, ...
76	6	<u>18</u>	<u>18</u>	<u>13</u>	<u>13</u>	<u>5.5</u>	<u>5.5</u>	4, 4, ...
78	6	<u>21</u>	<u>18</u>	<u>15</u>	<u>14</u>	<u>10</u>	<u>5</u>	4, 3, ...
80	6	<u>21</u>	<u>17</u>	<u>16</u>	<u>14</u>	<u>8</u>	<u>6</u>	4, 4, ...
82	6	<u>21</u>	<u>21</u>	<u>14.5</u>	<u>14.5</u>	<u>10.5</u>	<u>10.5</u>	4, 3, ...
84	5		<u>22</u>	<u>18</u>	<u>15</u>	<u>14</u>	<u>8</u>	4, 3, 3, ...
86	6	<u>24</u>	<u>22</u>	<u>19</u>	<u>14</u>	<u>14</u>	<u>10</u>	4, 3, ...
88	5		<u>47</u>	<u>19</u>	<u>19</u>	<u>12.5</u>	<u>12.5</u>	4, 3, 3, ...
90	5		<u>25</u>	<u>22</u>	<u>19</u>	<u>15</u>	<u>13</u>	3, 3, 3, ...
92	5		<u>25</u>	<u>24</u>	<u>19</u>	<u>15</u>	<u>12</u>	4, 3, 3, ...
94	5		<u>53</u>	<u>22.5</u>	<u>22.5</u>	<u>15.5</u>	<u>15.5</u>	3, 3, 3, ...
96	4		<u>25</u>	<u>24</u>	<u>19</u>	<u>15</u>	<u>15</u>	3, 3, 3, 3, ...
98	5	<u>27</u>	<u>26</u>	<u>24</u>	<u>18</u>	<u>16</u>		3, 3, 3, ...
100	4		<u>24.5</u>	<u>24.5</u>	<u>20.5</u>	<u>20.5</u>		3, 3, 3, 3, ...
102	4		<u>28</u>	<u>25</u>	<u>24</u>	<u>18</u>		3, 3, 3, 3, ...
104	4		<u>29</u>	<u>24</u>	<u>24</u>	<u>21</u>		3, 3, 3, 3, ...
106	4		<u>29.5</u>	<u>29.5</u>	<u>23</u>	<u>23</u>		3, 3, 3, 3, ...
108	3			<u>29</u>	<u>25</u>	<u>23</u>		4, 3, 3, 3, 3, ...
110	4		<u>33</u>	<u>29</u>	<u>25</u>	<u>23</u>		4, 3, 3, 3, ...
112	3			<u>60</u>	<u>25.5</u>	<u>25.5</u>		4, 4, 3, 3, 3, ...
114	3			<u>33</u>	<u>29</u>	<u>25</u>		4, 4, 3, 3, 3, ...
116	3			<u>33</u>	<u>28</u>	<u>26</u>		4, 4, 4, 3, 3, ...
118	3			<u>60</u>	<u>30.5</u>	<u>30.5</u>		4, 4, 4, 3, 3, ...
120	2			<u>33</u>	<u>33</u>	<u>28</u>		4, 4, 4, 4, 3, 3, ...
122	3			<u>31.5</u>	<u>31.5</u>	<u>30</u>		4, 4, 4, 4, 3, ...
124	2				<u>31</u>	<u>31</u>		4, 4, 4, 4, 4, 3, ...
126	2				<u>32</u>	<u>31</u>		4, 4, 4, 4, 4, 3, ...
128	2				<u>32</u>	<u>31</u>		4, 4, 4, 4, 4, 4, ...
130	2				<u>32.5</u>	<u>32.5</u>		4, 4, 4, 4, 4, 4, ...
132	1					<u>32</u>		4, 4, 4, 4, 4, 4, 3, ...
134	2				<u>35</u>	<u>33</u>		4, 4, 4, 4, 4, 4, ...
136	1					<u>70</u>		4, 4, 4, 4, 4, 4, 3, ...
138	1					<u>35</u>		4, 4, 4, 4, 4, 3, 3, ...
140	1					<u>37</u>		4, 4, 4, 4, 4, 4, 3, ...
142	1					<u>77</u>		4, 4, 4, 4, 4, 3, 3, ...
144	0							4, 4, 4, 4, 4, 3, 3, 3, ...
146	1					<u>39</u>		4, 4, 4, 4, 4, 3, 3, ...
148	0							4, 4, 4, 4, 4, 3, 3, 3, ...
150	0							4, 4, 4, 4, 3, 3, 3, 3, ...
152	0							4, 4, 4, 4, 4, 3, 3, 3, ...
154	0							4, 4, 4, 4, 3, 3, 3, 3, ...
156	0							4, 4, 4, 4, 3, 3, 3, 3, ...