

MA150/WR150
Investigations in Geometry
Spring 2018
Lectures: Tuesday, Thursday 11-12:15 in CAS 221
Discussion section: Tuesday 3:35-4:25 in CAS 114A

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Office Hours: Wednesday 11-12, 1-3 or by appointment

Course Website: Blackboard via learn.bu.edu. Be sure to check Blackboard regularly for course announcements.

Course Description

WR 100 and WR 150 make up a two-semester sequence of writing courses required of most Boston University undergraduates. They are designed to help all students acquire skills and habits of mind essential both to their academic success and to their future personal, professional, and civic lives. WR 100 and WR 150 are taught as small, topic-based seminars. Different sections of these courses address a range of different topics.

The specific topic of this section of WR 150 is developing proof writing skills and general mathematical writing skills through the study of Euclidean and non-Euclidean geometries. In particular, the mathematical content of this course includes axiomatics of geometry and a detailed study of geometries, including Euclidean geometry in the plane and space, real and complex projective geometry, the hyperbolic plane, and finite geometries. The key tool to investigating these geometries is the symmetry group of each geometry. Applications include quadratic equations in projective space, which gives a unified treatment of the ellipses, parabolas and hyperbolas studied in high school geometry, and an introduction to the mathematics of perspective drawing. The major theme of the course is to understand how Euclidean geometry fits in with these other, more exotic geometries.

Prerequisites:

The only prerequisite is a solid background in high school algebra, geometry and trig. Students may not take MA150 after having taken MA528.

Course Goals

Although they vary in topic, all sections of WR 100 and WR 150 have certain goals in common. In WR 100, you developed your abilities to:

- craft substantive, motivated, balanced academic arguments
- write clear, correct, coherent prose
- read with understanding and engagement
- plan, draft, and revise efficiently and effectively
- evaluate and improve your own reading and writing processes
- respond productively to the writing of others
- express yourself verbally and converse thoughtfully about complex ideas.

In WR 150, you will continue developing all of these abilities while working intensively on prose style and learning to conduct college-level research.

Course Requirements

As a writing seminar, WR 150 requires both a good deal of reading and writing and your active involvement in a variety of class activities. Specific course requirements are:

- self-assessment

- additional exercises as assigned
- three major papers
- final portfolio
- one conference with your instructor
- attendance and participation

Course Materials

Turabian, Kate L., Gregory G. Colomb, and Joseph M. Williams. *Student's Guide to Writing College Papers*. Chicago: University of Chicago, 2010. ISBN-13: 978-0-226-81631-9.

Lecture notes available on Blackboard (learn.bu.edu)

The online grammar handbook *Norton/Write* at

<http://www.wwnorton.com/college/english/write/writesite/>.

Optional text: Brannan, David A., Esplen, Matthew F., and Gray, Jeremy J. *Geometry*. Cambridge: Cambridge University Press, 2012.

Assignments

Style of the course: The main goal of the course is to convey some sense of how mathematicians discover new mathematics. The only way to do this is to experience some of the frustration and excitement of figuring things out for oneself. Consequently, new material will be introduced mostly through homework problem sets, which will ask for examples, counterexamples, generalizations, conjectures, and proofs. Class time will be divided between working through homework examples, singly or in groups, and more traditional lectures on the new material. The function of the lectures will be to compare the examples, solutions and hypotheses formulated by class members, and then to extract material that takes the class as a whole to the next level of understanding.

The writing component of the course is interwoven with the math component. Regular homeworks should have clear exposition and proofs in line with standard mathematical practices; these will be explained during the course. The three papers must exhibit a high content level, backed by standard writing practices on format, language, punctuation, grammar, and references, and combined with practices specific to mathematics, including logical organization of material, clearly displayed equations, worked examples, statements of results with proofs, further conjectures, etc. This will be gone over in detail in class.

By the end of the course, students should have a firm understanding of how to *write mathematical proofs* and how to *write both discursively and technically about mathematical topics*.

The Three Papers: This course counts as a Writing Program seminar. As such, students will prepare both standard math homework assignments and writing assignments. The major writing assignments will be:

Paper 1: a five- to seven-page paper on different approaches to math education;

Paper 2: a five-page paper for a general audience either on the student's geometric topic or on a general area of university mathematics such as calculus or linear algebra;

Paper 3: a ten-page report on students' investigations into a geometric topic chosen in consultation with the professor.

For Paper 1, some relevant articles are posted on Blackboard. For Papers 2 and 3, a list of possible projects will be posted on Blackboard, and students will consult with the professor to choose a topic. For Paper 3, you may choose to work in groups of up to three students; each student will receive the same grade for the paper.

Drafts and final versions: You will write a draft for each paper. Drafts will not receive explicit grades, although you will receive credit for completing them on time. Remember that you are

more likely to write a better final paper if you write a substantive draft before seeking peer or instructor feedback. All drafts and final papers must be word-processed and documented in MLA style. Please: no tricks with wide margins.

For the final project LaTeX is preferred. There are many online tutorials with sample files you can find by Googling “Latex tutorial.” TeXShop, a free package for writing and compiling Latex files on Macs, is available at pages.uoregon.edu/koch/texshop. For PCs, try TeXworks at tug.org/texworks/.

Oral Presentation: Near the end of the semester, you will give an oral presentation of around 10 minutes on the topic of your final paper. In this presentation, you will explain to the class the topic and methodology of our class capstone project. This presentation is not a formal speech, but a talk, after which you will receive feedback from your classmates.

Sharing of Student Writing: Experienced writers routinely share their work with others, because they understand that the best way to improve a piece of writing is to test it out with actual readers. In this class, you will learn how to respond productively to the writing of others and how to use feedback from others to improve your own work. All students in the class will be required to share at least one draft of each paper. If you are concerned about sharing your writing, please talk with me.

Homework: Homework sets will be assigned every week, and will be due one week after they are assigned. The homework sets contain a mixture of computational problems, problems requiring proofs, and problems requiring experimentation and formulating conjectures. You are not expected to work on every problem on every homework set, but you must hand in a selection of problems of different types. You may work with other students on the homework, but you must write up your own homework sets. I do not encourage you to use the web or other books for the homework. In any case, you **MUST** cite any books or websites you use to help with your homework; see the section below on plagiarism.

Grading and Evaluation

Your final grade will be calculated as follows:

Homework:	50%
Paper 1:	10%
Paper 2:	10%
Paper 3:	30%

You are welcome and encouraged to work with others on the homework.

Late and Missed Assignments: Late homework will not be accepted except for legal reasons such as jury duty, substantiated illness, family emergency or religious reasons.

Participation and Attendance: Under ordinary circumstances, missing more than one week of class will lower your final grade. Missing more than two weeks of class may lead to a failing grade in the course. Note that these absences need not be consecutive. If you have a special obligation that will require you to miss several classes (e.g., religious observances), please talk with me at the beginning of the semester.

CAS Center for Writing Writing Resources

At the CAS Center for Writing (100 Bay State Road, 3rd floor with a satellite office at Mugar Library) students enrolled in WR courses can receive one-on-one consultations about their writing with well-trained tutors familiar with WR assignments. When you visit the center, you should expect to be actively involved in your session. Tutors will work with you at any stage in your

writing process, but they will not edit or correct your paper for you. Rather, they will work with you to help you do your own best work. The center is a resource for all WR students. Whether you consider yourself to be a strong writer or a weak one, you can benefit from consulting with a tutor.

The CAS Center for Writing is open Monday through Friday. Hours for the current semester are posted on the website below (common hours are between 9 a.m. and 5:15 p.m. or 7:30 p.m.). While the center accepts walk-in visits, you are strongly encouraged to make an appointment in advance. Because of the high demand for consultations, students are limited to one reservation per week. You may schedule a session online:

<http://www.bu.edu/writingprogram/the-writing-center/>

You may also schedule a session in person at the CAS Center for Writing or by calling 617-358-1500. Cancellations must be made at least 12 hours in advance.

WR

The CAS Writing Program publishes an online journal of exemplary writing from WR courses. If you are interested in looking at samples of successful WR papers, or if you just want to read some good essays, I encourage you to visit the journal:

<http://www.bu.edu/writingprogram/journal/>

Plagiarism

Plagiarism is the passing off of another/s words or ideas as your own, and it is a serious academic offense. Cases of plagiarism will be handled in accordance with the disciplinary procedures described in the College of Arts and Sciences Academic Conduct Code. All WR students are subject to the CAS code, which can be read online:

<http://www.bu.edu/academics/resources/academic-conduct-code/>

Penalties for plagiarism can range from failing an assignment or course to suspension or expulsion from the university. In this class, we will discuss conventions for using and citing sources in academic papers. If you have any questions about plagiarism, I invite you to speak with me.

Plagiarism in mathematics has special issues, as it can occur in homework as well as in the papers. In particular, if you use a website for a homework problem, you must reference it in a footnote or in a bibliography. However, if you use a standard math fact like $ab = ba$ that you have looked up in a text or on the web, it need not be cited. This is a grey area: if you're unsure whether or not to cite a reference, just cite it!

Rough Outline of Contents

Th 1/18: Review of vectors in the plane \mathbb{R}^2 and in space \mathbb{R}^3 , manipulations with complex numbers, multiplication of 2×2 and 3×3 matrices, and relationships between e.g. complex multiplication and matrix multiplication. Discussion of finite geometries and their symmetries.

Tu 1/23: Continuation of 1/18. *Paper 1 on math ed issues discussed and assigned.*

Th 1/25: The axiomatic/synthetic approach to geometry vs. the analytic approach. Euclid's axioms and Hilbert's axioms. *Overview and discussion of writing projects (Paper 3).*

Tu 1/30: See 1/25.

W 1/31: Last day to add or change WR courses

Th 2/1: The parallel postulate in Euclidean and non-Euclidean geometries. The definition of a group. *First draft of Paper 1 due.*

Tu 2/6: See 2/1. Peer review of Paper 1 due. *Selection of Paper 3 writing project finalized.*

Th 2/8: The symmetry group of a geometry: examples of symmetries of \mathbb{R}^2 , polygons, \mathbb{R}^3 , polyhedra and finite geometries.

Tu 2/13: See 2/8.

Th 2/15: The symmetry group of \mathbb{R}^2 . *Final version of Paper 1 due. Paper 2 assigned.*

Tu 2/20: No class! Substitute Monday schedule.

Th 2/22: Last day to drop without a “W” grade.

Th 2/22: See 2/15.

Tu 2/27: The real and complex projective planes $\mathbb{RP}^2, \mathbb{CP}^2$; synthetic and analytic aspects. *First draft of Paper 2 due.*

Th 3/1: See 2/27.

Tu 3/6, Th 3/9. Tu 3/13, Th 3/15: **No class!**

Th 3/20: The real projective plane and perspective drawing. Desargue’s Theorem.

Tu 3/22: See 3/20.

Th 3/30: The symmetry group of \mathbb{RP}^2 . *Final version of Paper 2 due.*

Th 3/30: Last day to drop with a “W” grade

Tu 4/3: See 3/30.

Th 4/5: Affine planes and projective planes over $\mathbb{R}, \mathbb{C}, \mathbb{Z}_p$.

Tu 4/10: See 4/5. *First draft of Paper 3 due.*

4/11: Last day to officially take a leave of absence or withdraw from the University for Spring 2017.

Th 4/12: Embedding affine planes in projective planes. Moving lines to infinity: using projective geometry to prove theorems in Euclidean geometry.

M 4/16: Holiday, Classes Suspended.

Tu 4/17: Conics in \mathbb{R}^2 : their classification, relation to quadratic equations, and their equivalence via projective geometry.

W 4/18: Substitute Monday Schedule of Classes.

Th 4/19: See 4/17.

Tu 4/24: Intersecting conics in $\mathbb{R}^2, \mathbb{RP}^2, \mathbb{C}^2, \mathbb{CP}^2$.

Th 4/26: The number of degree d curves on $3d-1$ points in \mathbb{CP}^2 : an introduction to Kontsevich’s theorem. Student presentations.

Tu 5/1: See 4/26. *Final version of Paper 3 due.* Student presentations. Course evaluations.