

Solving a Linear System

$$\begin{aligned}2x_1 + 3x_2 - x_3 &= 1 \\x_1 + 0x_2 - x_3 &= 4 \\-x_1 + 2x_2 + 2x_3 &= -9\end{aligned}$$

$$\begin{aligned}x_1 + 0x_2 - x_3 &= 4 & \mathbf{(1)} \\2x_1 + 3x_2 - x_3 &= 1 & \mathbf{(2)} \\-x_1 + 2x_2 + 2x_3 &= -9 & \mathbf{(3)}\end{aligned}$$

$$\begin{aligned}x_1 + 0x_2 - x_3 &= 4 \\0x_1 + 3x_2 + x_3 &= -7 & [-2(\mathbf{1}) + (\mathbf{2})] \\0x_1 + 2x_2 + x_3 &= -5 & [(\mathbf{1}) + (\mathbf{2})]\end{aligned}$$

$$\begin{aligned}x_1 + 0x_2 - x_3 &= 4 \\0x_1 + x_2 + \frac{1}{3}x_3 &= -\frac{7}{3} & \left[\frac{1}{3}(\mathbf{2})\right] \\-x_1 + 2x_2 + 2x_3 &= -9\end{aligned}$$

Repeating the previous system:

$$\begin{aligned}x_1 + 0x_2 - x_3 &= 4 \\0x_1 + x_2 + \frac{1}{3}x_3 &= -\frac{7}{3} \\-x_1 + 2x_2 + 2x_3 &= -9\end{aligned}$$

$$\begin{aligned}x_1 + 0x_2 - x_3 &= 4 \\0x_1 + x_2 + \frac{1}{3}x_3 &= -\frac{7}{3} \\0x_1 + 0x_2 + \frac{1}{3}x_3 &= -\frac{1}{3} \quad [-2(\mathbf{2}) + \mathbf{3}]\end{aligned}$$

Now solve from the bottom up:

$$x_3 = -1$$

$$x_2 = -2$$

$$x_1 = 3$$