MA725–Differential Geometry I

Fall 2014

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Text: "The Laplacian on a Riemannian Manifold," Cambridge U. Press, available on my webpage.

Grading: From assigned homework problems

Differential geometry is the study of smooth manifolds with the additional data of a Riemannian metric. This breaks into the study of (i) local properties of metrics, i.e. curvature and covariant differentiation, (ii) global properties such as the existence of geodesics, and (iii) local/global questions of the form "Which manifolds admit metrics with certain curvature conditions?" These topics are related by studying analysis on manifolds. In particular, Laplacian-type second order differential operators on Riemannian manifolds play a crucial role. The study of the heat flow associated to these operators establishes a precise relationship between the kernels of Laplacians and the topology of the underlying manifold (Hodge theory), and gives some local/global results. Some of this carries over to vector bundles with connections, giving a geometric approach to characteristic classes.

In more detail, in Chapter I the heat equation approach to Hodge theory will be discussed. The complete picture depends on a minimal understanding of Riemannian geometry, which is the subject of Chapter II. The main topics in this chapter are: curvature and accurate mappings, geodesics and Jacobi fields, and supersymmetric Weitzenböck formulas for Laplacians. In Chapter III, these geometric techniques are used to construct the heat kernels for the Laplacians on forms, which finally completes the Hodge theory picture. Chapter IV covers a heat equation proof of the Chern-Gauss-Bonnet theorem, relating the Euler characteristic of a closed manifold to curvature information. Via the Chern-Weil theory of characteristic classes, this approach generalizes to the Atiyah-Singer index theorem.