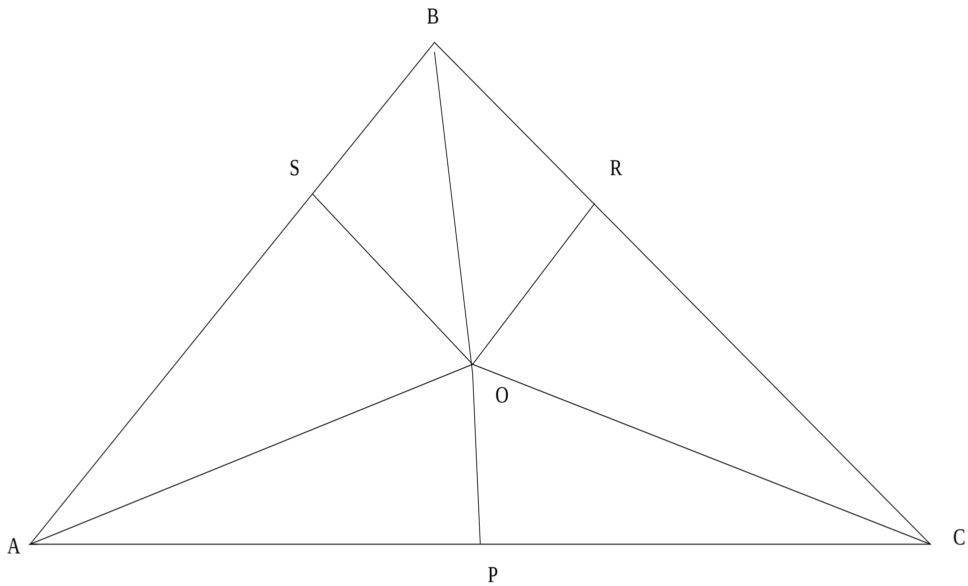
All Triangles are Isosceles

What is wrong with the following proof that all triangles ΔABC are isosceles?



Draw the bisector of $\angle ABC$ and the perpendicular bisector of side AC. These meet at O. Construct OS and OR perpendicular to AB and BC, respectively. Draw OA and OC.

Since $|AP| = |PC|$ and $\angle OPA, \angle OPC$ are right angles, $\Delta APO \cong \Delta CPO$ are congruent by SAS. Thus $|AO| = |CO|$.

ΔBOS and ΔBOR are right triangles with the same hypotenuse and $\angle OBS = \angle OBR$. Call this angle θ. Then $|SB| = |BO| \sin(\theta) = |RB|$. By the Pythagorean theorem, $|OS| = |OR|$. (Alternatively, to skip the trig, these two triangles have corresponding anlges equal and hence are similar. Since they share a side, they are congruent, so $|OS| = |OR|$.)

ΔAOS and ΔCOR are right triangles with hypotenuses AO and CO of the same length, and with sides OS and OR of the same length. Again by the Pythagorean theorem, $|AS| = |CR|$.

Therefore, $|AB| = |AS| + |SB| = |CR| + |RB| = |BC|$, and ΔABC is isosceles.

Adapted from `euler.slu.edu/Dept/Faculty/clair/math/puzzlers.html`