**ST112**
**Notes for Week V**
**Spring 2011**

### Trig functions – sine and cosine

Sin and Cos convert circular motion to linear motion. Physical examples of sin: (i) looking at the height of a red dot on the edge of a spinning unit radius circular disk as a function of time – in particular, think of the height of a piston as a function of time; (ii) plotting the amplitude of a sound/electomagnetic wave a function of time. Physical examples of cos: (i) same as the first example above, but looking at the x-displacement of the red dot as a function of time; (ii) as sin with a "lag time of $\pi/2$."

The periodicity of sin and cos – why do we measure radians the way we do?

The role of amplitude and frequency/period: $A\cos(k\theta)$ has amplitude $A$, period $2\pi/k$, frequency $k/2\pi$. $A\cos(k\theta - \psi)$ has a phase shift/lag time of $\psi$.

### Trig functions – tangent

$\tan\theta = \sin\theta/\cos\theta$. If a line $y = mx + b$ cuts the x-axis making an angle $\theta$ with the positive x-axis, then $\tan\theta = m$.

### Inverse trig functions

Arcsin or $\sin^{-1}$ answer the question: If my red dot is at height $y$, what is the angle formed by a horizontal line, the center of the disk, and the dot? Arccos or $\cos^{-1}$ answers the same question for the x-displacement. Arctan or $\tan^{-1}$ answers the question: What is the angle $\theta$ that a line with slope $m$ makes with the x-axis?

What are (good) domains/ranges for $\sin^{-1}, \cos^{-1}, \tan^{-1}$?

### Trig formulas

Which ones are important?
$|\cos\theta| \leq 1; |\sin\theta| \leq 1$.
$\cos(\theta + 2\pi) = \cos\theta; \sin(\theta + 2\pi) = \sin\theta$.
$\cos^2\theta + \sin^2\theta = 1$.
$\cos(\theta - \pi/2) = \sin\theta; \sin(\theta + \pi/2) = \cos\theta$.
The addition formulas

$$\cos(\theta + \psi) = \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi);\ \sin(\theta + \psi) = \cos(\theta)\sin(\psi) + \sin(\theta)\cos(\psi).$$

Note that a number on the unit circle has coordinates $(\cos\theta, \sin\theta)$ for some $\theta$. Write this as $\cos\theta + i\sin\theta$. Check that $(\cos\theta + i\sin\theta)\cdot(\cos\psi + i\sin\psi) = \cos(\theta + \psi) + i\sin(\theta + \psi)$. So complex multiplication codes up the addition formulas.