ST112
Notes for Week VI
Spring 2011

Calculus: motivation

Toy models of physical (e.g. planetary) motion: (i) graphs of position vs. time for constant speed motion on a line; (ii) graphs of position vs. time for variable speed motion on a line.

The basic questions: (a) How can you read off the velocity of the particle at a particular time from its graph? (b) Conversely, if you can measure/graph the velocity (as with a speedometer), how can you read off the total distance traveled by a particular time?

Secondary issues: (c) Distinguish speed from velocity; (d) distinguish total distance traveled from net distance traveled.

Differentiation

This answers (a), computing the velocity of the particle. For constant speed motion, the slope of the straight line graph is the velocity. For variable speed motion, the slope of small secant lines approximates the velocity, "so" the slope of the tangent line must be the velocity.

Reading off velocity graphs from position graphs. How can you tell when the velocity is negative/positive/zero? Relation to max/min problems.

Reading off position graphs from velocity graphs. Reading off acceleration graphs from velocity graphs. Reading off velocity graphs from acceleration graphs.

The role of limits in calculus. Informal vs. formal understanding of limits. Why we forbid the language of infinitesimals. Continuous vs. differentiable functions. What does the graph of a continuous function look like? What does the graph of a discontinuous function look like? Can a function $f : [0, 1] \to [0, 1]$ be continuous at an infinite number of points and discontinuous at an infinite number of points?

Computations of derivatives: this is (half the reason) why calculus is called calculus. We can compute derivatives of polynomials, trig, exponential and log functions. We can compute derivatives of compositions of these functions by the Chain Rule.

What happens to calculus if we use degrees rather than radians?

Estimates using derivatives

How can we extrapolate an output of an experiment if the input is not available, but nearby inputs are? How can we estimate the error in our extrapolation?

Integration

This answers (b), computing the (net) distance traveled by the particle. For constant speed motion, the net distance is the area of the box bounded by the position graph and the x-axis. For the total distance, we have to use the graph for the absolute value of the position graph. For variable speed motion, we approximate the graph by a sequence of constant velocity graphs, and naively "take a limit." This is the integral.

Indefinite vs. definite integrals. A difficult theorem: continuous functions can be integrated. After polynomials, computing integrals directly becomes too difficult.

The fundamental theorem of calculus

A careful statement of the two fundamental theorems. Sketch of the proof. Now if we can explicitly differentiate a function f, we can explicitly integrate the derivative f'. Calculus books are filled with integral problems. What "precentage" of functions can we really integrate?

Re-reading position graphs from velocity graphs, and velocity graphs from acceleration graphs.