ST112, Problem Set 4
Spring 2011

Estimation and calculation

In these problems, make an educated guess. Then calculate the answer exactly if possible. If an exact answer isn't possible, you can use a graphing calculator or any graphing software to make good estimates. If an exact or very accurate answer is possible, note how close or far your guess was from the exact answer on your write up (and of course write up the exact answer). If only an estimate is possible, explain how you got your estimate, and estimate the margin of error.

For P5, we'll tabulate all your guesses and calculate the mean (= average) and median. Then we'll compare the mean and median to the actual answer, to see how well you collectively do with educated guesses.

P1. A glass jar in the shape of a cylinder with radius 5 inches and height one foot is filled with jellybeans. How many jellybeans are in the jar? (Don't forget that the jellybeans don't pack completely tightly.)

P2. For the same glass jar, how many grains of rice can fit in the jar?

P3. If you divide your answer in P2 by your answer in P1, do you get an accurate estimate of the number of grains of rice that fit in one jellybean? Why or why not?

P4. (Be sure to make a guess before you calculate.) Look up the circumference of the earth and the moon. Place a steel band one foot above the surface of the earth going around the equator. (Don't ask why.) How much longer than the earth's circumference is the steel band? Do the same thing for the moon.

P5. A fictitious professor makes $75,000 per year and pays $100 per month for parking. Her salary goes up 2.5% per year, and the parking rate goes up 4% per year. In how many years will her salary equal her parking costs?

P6. Parking rates are jacked up to a 10% increase per year, while the salary increases stay at 2.5%. The professor decides she'll retire when her salary minus her parking costs is at a maximum. How many years until she retires?

P7. I want to borrow $5000. Bank A will offer me a flat interest rate of 2.5% compounded annually. Bank B offers the following deal: for the first 10 years, the loan is interest free, but there is a "processing fee" of $500 payable when the loan is taken out. After 10 years, the bank starts charging a rate of 4% compounded annually on the $5000. Which loan is better if I want to pay off the loan after 2 years? Which is better for a 10 year loan? A 20 year loan? A 25 year loan? At which time(s) are the two loans equally good?

Trig problems

P8. a. A garden hose is stored on a circular wheel of radius one foot. If the hose is 100 feet long when fully stretched out to the right and attached to the right most point on the wheel, how many times will the wheel turn around to wind up the hose? (Neglect the thickness of the hose.)

b. After the hose is wound up, what angle is formed by the tip of the hose, the center of the wheel, and the horizontal axis going through the center of the wheel an pointing to the right?

P9. The same problem as P8a, but now assume that the hose can wind five times around the wheel before the hose has to lie on top of a previous layer of hose. Assume that the hose has a radius of 1/2 inch.

P9. A CD rotates at about 500 rpm (revolutions per minute) at its inner edge and about 200 rpm at its outer edge.[1] Measure the distance to the inner and outer edges of a CD. Compute how fast (in miles per hour) a point on the inner edge is moving, and compute how fast a point on the outer edge is moving.

P10. While most car engines run at about 2500 rpm in normal driving conditions, Formula One racing car engines can run up to 18,000 rpm.[2] How far does the top of a piston attached to a crankshaft with a radius of 4 inches travel in one second for an engine running at 18,000 rpm? (A picture of a crankshaft and pistons is at

`http://jordanalleycat.files.wordpress.com/2010/07/crankshaft_pistons.jpg`.

The circular bar at the bottom of the picture rotates at 18,000 rpm, causing the pistons to move up and down; the radius refers to the distance from the center of the circular rod to the edge of the fat part of the key shaped rotating piece attached to the circular rod.)

Challenge Problems

P11. Show that the points (y, x) and (x, y) are reflections of each other across the line $y = x$. You can prove this using some trig, but see if you can give a proof just using high school algebra. (We used this in class to draw the inverse function f^{-1} for a function $f : \mathbb{R} \to \mathbb{R}$.)

P12. Show that $\cos \pi/9 = \cos 20^o$ is the root of a cubic polynomial $p(x)$ with integer coefficients. (Hint: derive a "triple angle formula" for angles, and note that $\cos \pi/3 = 1/2$.) Show that this polynomial does not factor into a linear and quadratic term, each with integer coefficients.

Note: This is the key step to showing that it is impossible to trisect a 60^o angle using only a straightedge and compass; see e.g. A. Baragar, *A Survey of Classical and Modern Geometries: With Computer Activities*, Pearson, Boston, 2001. In contrast, angle trisection is possible in origami; see
`http://www.cut-the-knot.org/pythagoras/PaperFolding/index.shtml` and
`http://www.cut-the-knot.org/pythagoras/PaperFolding/AngleTrisection.shtml`.

[1] `http://en.wikipedia.org/wiki/Revolutions_per_minute`
[2] Wikipedia, *op. cit.*