ENG EK 103: Computational Linear Algebra: Problem set 1

Due: Thursday, 09.12.2019 (A1-A3) OR Monday, 09.16.2019 (A4)

On your submitted papers, be sure to write down your name, BU ID, lecture section (A1-4), and the number of the homework. Problems that do not specifically involve Matlab should be solved by hand.

1. Suppose $\boldsymbol{u} = \begin{bmatrix} 3 & -1 & 1 \end{bmatrix}^T$, $\boldsymbol{v} = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}^T$, and $\boldsymbol{w} = \begin{bmatrix} -2 & -1 & -3 \end{bmatrix}^T$.
   (a) Calculate $\boldsymbol{u} \cdot \boldsymbol{v}$ and $\boldsymbol{u} \cdot \boldsymbol{w}$.
   (b) Calculate $\boldsymbol{x} = \boldsymbol{v} + \boldsymbol{w}$, then calculate $\boldsymbol{u} \cdot \boldsymbol{x}$, and finally verify it equals $(\boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \cdot \boldsymbol{w})$.
   (c) Show that this *distributive* property holds for any 3-D vectors (that is for any $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^3$).
2. Consider the two vectors $\boldsymbol{u} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$ and $\boldsymbol{v} = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}^T$.
   (a) Find a and b such that $a\boldsymbol{u} + b\boldsymbol{v} = \begin{bmatrix} 3 & -1 & -2 \end{bmatrix}^T$.
   (b) Explain why there is no choice of a and b that satisfies $a\boldsymbol{u} + b\boldsymbol{v} = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}^T$.
3. Calculate the angle between the following pairs of vectors
   (a) $\boldsymbol{u} = \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}^T$ and $\boldsymbol{v} = \begin{bmatrix} 0 & -4 & 0.25 \end{bmatrix}^T$.
   (b) $\boldsymbol{u} = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \end{bmatrix}^T$ and $\boldsymbol{v} = \begin{bmatrix} 2 & 0.36 & 2 & 4 & \pi \end{bmatrix}^T$.
4. Download and install Matlab on your computer. See:

   http://www.bu.edu/tech/support/research/software-and-programming/common-languages/matlab/

   Read through the tutorial at:

   https://www.mathworks.com/help/matlab/getting-started-with-matlab.html?s_cid=learn_doc

   Please note that we do not expect prior experience with Matlab and for the most part you will be running m files (programs) that have been written for you. Please feel free to make heavy use of your GSTs to help you! You may also find questions posted on Slack to be useful.
5. Consider the plot below with three data points on the $x - y$ plane.
   (a) Find vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ that specify the positions of the data points A, B, C with respect to the origin.
   (b) Calculate the angles between $\boldsymbol{u}$ and $\boldsymbol{v}$ (θ_1), between $\boldsymbol{v}$ and $\boldsymbol{w}$ (θ_2), and between $\boldsymbol{w}$ and $\boldsymbol{u}$ (θ_3).
   (c) Calculate the sum $\theta_1 + \theta_2 + \theta_3$.
   (d) Download the m-file `ps1_prob4.m` from slack and read it to understand the procedures. Then run the file, print out the plot, and attach it to your homework submission.
   (e) Using the plot, explain why your answer to (c) makes sense.