MA 123 - Practice Exam #2

Name:

Instructions: For some of the questions, you must show all your work as indicated. No calculators, books or notes of any form are allowed.

Note that the questions have different point values. Pace yourself accordingly.

Good luck!

Question	Score	Out of
1		60
2		15
3		15
4		20
5		15
6		40
7		30
8		18
9		20
Total		233

1. (**60 points**). Differentiate the following functions:

a) $f(x) = \exp(x^2 + 1)$

b) $f(x) = \frac{sin(2x)}{e^{2x}-1}$

c) $f(x) = x^2 ln(cos^2(x) - 1)$

d) $f(x) = \frac{3^x tan(x)(x+1)^{10}}{(x^2-1)(x+2)}$

e) $f(x) = x^{\sqrt{x}}$

f) $f(x) = arcsin(3x - 1)$

2. **15 points**. Find the equation of the tangent line to the curve $x^{2/3} + y^{3/5} = 2$ at the point $(1, 1)$.

3. (**15 points**) Evaluate

$$\lim_{n\to\infty}(1+\frac{2}{n})^n$$

4. (**20 points**)

A trough is 10 ft. long, and its ends have the shape of isoceles triangles that are 3 ft. across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min., how fast is the water level rising when the water is 6 inches deep ?

5. (**15 points**)

Find the absolute maximum and absolute minimum of the function

$$f(x) = x - \ln(x)$$

on the interval $[\frac{1}{2}, 2]$.

6. (**40 points**) Let $f(x) = \ln(x^4 + 27)$.

(a) Find the intervals of increase / decrease of $f(x)$.

(b) Find the local minima and maxima.

(c) Find the intervals of concavity and the inflection points

(d) Sketch the graph.

7. (**30 points**) Evaluate the following limits

a) $\lim_{x\to 1}(\frac{1}{ln(x)} - \frac{1}{x-1})$

b) $\lim_{x\to 0} \cot(2x)\sin(6x)$

c) $\lim_{x \to 0^+} x^{x^2}$

8. (**18 points.**)

Determine whether the following statements are true or false. If true, give a complete argument (referring to theorems, properties etc.). If false, give a counterexample.

(a) Suppose that f is a continuous function on the interval $[1, 3]$. A global max./min. of f always occurs at a critical number.

(b) Suppose that f is a continuous function on the open interval $(1, 2)$. Then there exist $1 < c < 2$ and $1 < d < 2$ such that $f(c)$ is the global maximum of f on $(1, 2)$ and $f(d)$ is the global minimum of f on $(1, 2)$.

(c) Suppose that f is a differentiable function on $(1, 2)$ that has a vertical asymptote at $x = 2$. Then both of the following statements cannot be true:

   i. f achieves an absolute minimum on $(1, 2)$
   ii. f achieves an absolute maximum on $(1, 2)$

9. (**20 points**)

In an automobile race along a straight road, car A passed car B twice. Prove that at some time during the race their accelerations were equal. (Prove means give a rigorous, concise, complete mathematical argument, referring to theorems etc. as needed).