

SOLUTIONS

MA 123 - Practice Exam #1

Name: _____

Instructions: For some of the questions, you must show all your work as indicated. No calculators, books or notes of any form are allowed.

Note that the questions have different point values. Pace yourself accordingly.

Good luck!

Question	Score	Out of
1		15
2		15
3		15
4		20
5		10
6		20
7		20
8		15
9		30
Total		160

1. (15 points). A particle is moving along a line. The distance it has traveled after t seconds is $s(t) = t^3 + 2t^2$ meters.

(a) What is the average speed of the particle over the interval $t = 1$ to $t = 2$?

$$\begin{aligned} V_{\text{avg}} &= \frac{s(2) - s(1)}{2 - 1} = \frac{(2^3 + 2 \cdot 2^2) - (1^3 + 2 \cdot 1^2)}{2 - 1} \\ &= 13 \text{ m/s} \end{aligned}$$

(b) Set up the limit that expresses the instantaneous speed of this particle at $t = 1$. You do not have to evaluate this limit.

$$\begin{aligned} v(1) &= \lim_{h \rightarrow 0} \frac{s(1+h) - s(1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[(1+h)^3 + 2(1+h)^2] - [1^3 + 2(1)^2]}{h} \end{aligned}$$

2. (15 points).

Let

$$f(x) = \begin{cases} e^{-x^2}, & \text{if } x < 0; \\ \sin(x), & \text{if } 0 \leq x \leq 2\pi; \\ x - 2\pi, & \text{if } x > 2\pi. \end{cases}$$

(a) Where is $f(x)$ continuous? Justify your answer.

e^{-x^2} is continuous on $(-\infty, 0)$ } The only question is
 $\sin(x)$ — u — $(0, 2\pi)$ } what happens at $x=0, 2\pi$
 $x-2\pi$ — u — $(2\pi, \infty)$

$$\underline{x=0}: \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} e^{-x^2} = e^{-0^2} = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \sin(x) = 0$$

$\Rightarrow \lim_{x \rightarrow 0} f(x)$ does not exist

$\Rightarrow f$ is not cont. at $x=0$

$$\underline{x=2\pi}: \lim_{x \rightarrow 2\pi^-} f(x) = \lim_{x \rightarrow 2\pi^-} \sin(x) = 0$$

$$\lim_{x \rightarrow 2\pi^+} f(x) = \lim_{x \rightarrow 2\pi^+} (x - 2\pi) = 0$$

$$f(2\pi) = 0$$

$\Rightarrow f$ is cont. at $x=2\pi$

f is cont. for all $x \neq 0$

(b) What is $\lim_{x \rightarrow -\infty} f(x)$? Justify your answer.

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} e^{-x^2} = \lim_{t \rightarrow -\infty} e^t = 0.$$

3. (15 points). Find the equation of the tangent line at $x = 2$ for the function $f(x) = \frac{x}{x+2}$. Show all work. Do not use derivative formulas not discussed in class.

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(2+h)}{(2+h)+2} - \frac{2}{2+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2+h}{4+h} - \frac{1}{2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(2+h) - (4+h)}{2(4+h)h} \\ &= \lim_{h \rightarrow 0} \frac{4+2h-4-h}{2(4+h)h} \\ &= \lim_{h \rightarrow 0} \frac{h}{2(4+h)h} \\ &= \lim_{h \rightarrow 0} \frac{1}{2(4+h)} = \frac{1}{2(4+0)} = \frac{1}{8} \end{aligned}$$

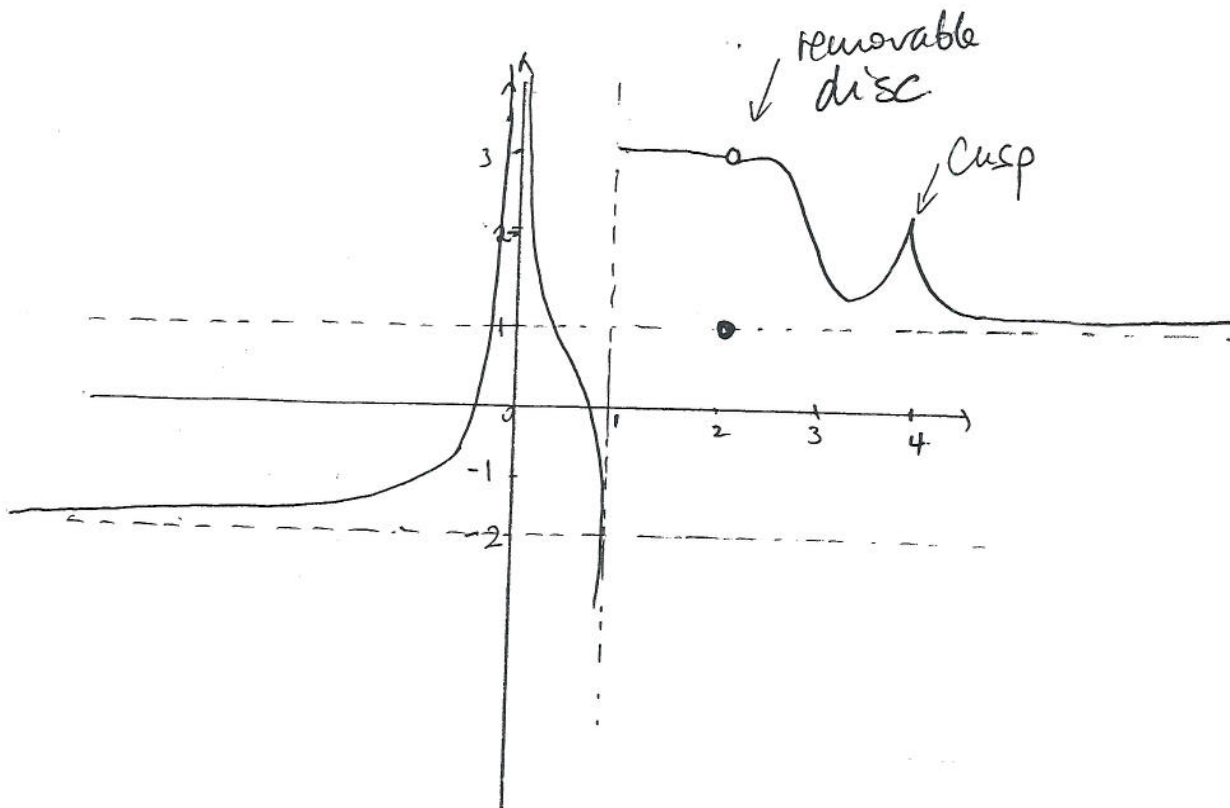
$f(2) = \frac{1}{2}$, so the line has slope $\frac{1}{8}$, passes through $(2, \frac{1}{2})$

$$(y - \frac{1}{2}) = \frac{1}{8} (x - 2)$$

4. (20 points). Sketch the graph of a function $f(x)$ satisfying the following properties:

- (a) $\lim_{x \rightarrow \infty} f(x) = 1$
- (b) $\lim_{x \rightarrow -\infty} f(x) = -2$
- (c) $\lim_{x \rightarrow 2} f(x) = 3$
- (d) $f(x)$ has vertical asymptotes at $x = 0$ and $x = 1$
- (e) $f(x) > 0$ for $x < 0$
- (f) $f(x)$ is discontinuous at $x = 2$.
- (g) $f(x)$ is not differentiable at $x = 4$.

typo →



5. (10 points). Find $\lim_{x \rightarrow \pi/2} \ln(\tan^2(x))$. Justify your answer.

$$\text{let } t = \tan^2(x).$$

$$\text{as } x \rightarrow \frac{\pi}{2}, \quad t \rightarrow \infty$$

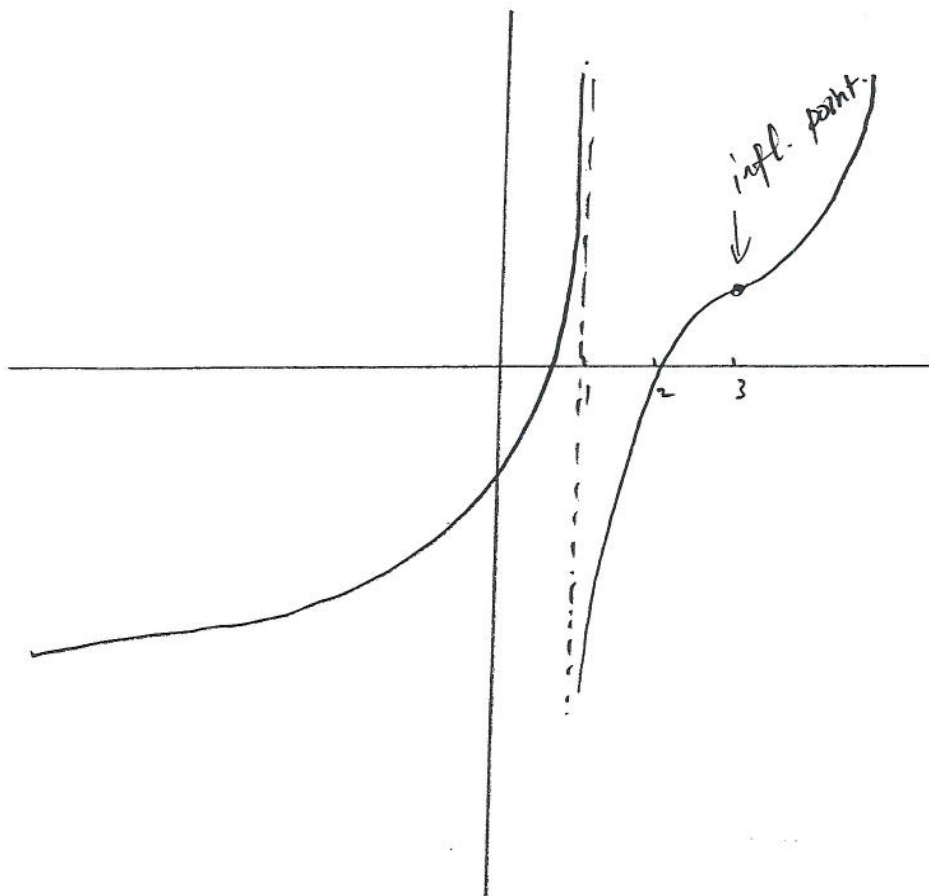
$$\lim_{x \rightarrow \frac{\pi}{2}} \ln(\tan^2(x)) = \lim_{t \rightarrow \infty} \ln(t) = \infty$$

6. (20 points). Find a formula for a function having vertical asymptotes at $x = -1$, $x = 1$, and $x = 3$ and a horizontal asymptote at $y = 2$.

$$f(x) = \frac{1}{(x+1)(x-1)(x-3)} + 2.$$

7. (20 points). Sketch the graph of a function $f(x)$ which satisfies all of the given conditions:

- (a) $f'(x) > 0$ for all $x \neq 1$
- (b) vertical asymptote at $x = 1$
- (c) $f''(x) > 0$ if $x < 1$ or $x > 3$
- (d) $f''(x) < 0$ if $1 < x < 3$



8. (15 points). Differentiate the function $f(x) = \sqrt[3]{x^2} - 2e^x$

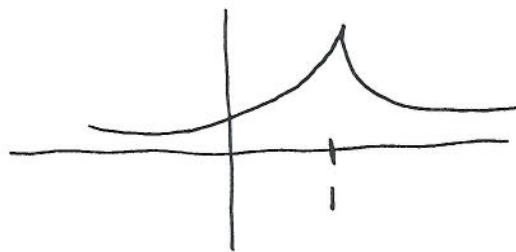
$$f(x) = x^{2/3} - 2e^x$$

$$f'(x) = \frac{2}{3}x^{-1/3} - 2e^x$$

9. (30 points). For each statement, determine whether it is *TRUE* or *FALSE* and write a sentence or two justifying your answer.

- (a) If $f(x)$ is continuous at $x = 1$, then it is differentiable at $x = 1$.
- (b) If $\lim_{x \rightarrow 0} f(x) = 2$ and $\lim_{x \rightarrow 0} g(x)$ does not exist, then $\lim_{x \rightarrow 0} [f(x) + g(x)]$ does not exist.
- (c) Let $f(x)$ be a continuous function such that $f'(x) > 0$ for $x > 2$. If $f(2) = 3$ then $f(3) > 3$.
- (d) A function can have infinitely many horizontal asymptotes.
- (e) The polynomial $x^3 + 3x - 1$ has a root between 0 and 1.

a) FALSE:



$f(x)$ is cont. at $x=1$
but not diff. at $x=1$

b) TRUE: if $\lim_{x \rightarrow 0} [f(x) + g(x)]$ exists, then

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} [f(x) + g(x)] - \lim_{x \rightarrow 0} [f(x)]$$

implying that $\lim_{x \rightarrow 0} g(x)$ exists, which is not true.

c) TRUE: $f'(x) > 0$ implies f is increasing,
so $f(3) > f(2) = 3$.

d) FALSE: it can have at most 2.

e) TRUE: let $f(x) = x^3 + 3x - 1$ so by
 f is continuous, and $f(0) = -1$ Int. val.
 $f(1) = 3$ then has a root in