MA 242 – Linear Algebra
Exam #1

Name:

Instructions: To receive full credit you must show all work. Explain your answers fully and clearly. You may refer to theorems/facts in the book or from class. No calculators, books or notes of any form are allowed. Good luck!

Question	Score	Out of
1		12
2		12
3		18
4		12
5		18
6		12
7		32
Total		116

1. **(12 points)**
Consider the system of equations

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6. \end{aligned}$$

Find the general solution to these equations in parametric form. What geometric shape does the solution space form?

2. **(12 points)**

Are the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix},$$

linearly independent?

(b) Give an example of 4 vectors in $\mathbf{R}^3$ that are linearly independent or explain (algebraically) why such an example does not exist.

3. **(18 points)**

(a) Define what it means for $T : \mathbf{R}^n \longrightarrow \mathbf{R}^m$ to be a linear transformation.

(b) Let $T : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ be defined by $T(x_1, x_2) = (x_1+x_2, x_1)$. Determine the matrix associated to T.

(c) Let $T : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ be the linear transformation defined by first rotating by $\pi/2$ radians counterclockwise and then reflecting over the x-axis. Determine the matrix associated to T.

4. **(12 points)**

Is $\mathbf{b}$ in the span of the columns of A where

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}?$$

Explain your answer.

(b) How many solutions (if any) does the equation $A\mathbf{x} = \mathbf{b}$ have where

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix}?$$

Explain your answer.

5. **(18 points)**

(a) Does there exist a map from $\mathbf{R}^4$ to $\mathbf{R}^7$ that is both one-to-one and onto? If yes, give an example. If no, explain why not.

(b) Does there exist a map from $\mathbf{R}^3$ to $\mathbf{R}^2$ that is onto, but not one-to-one? If yes, give an example. If no, explain why not.

(c) Does there exist a map from $\mathbf{R}^4$ to $\mathbf{R}^4$ that is one-to-one, but not onto? If yes, give an example. If no, explain why not.

6. **(12 points)**
Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ -2 & 1 \end{pmatrix}.$$

Only one of the products AB and BA makes sense. Determine which one and compute that product.

7. **(32 points)** Circle either TRUE or FALSE. No justification is needed. **Correct answers score 4 points each. Incorrect answers score 0 points. Leaving a question blank scores 2 points.**

(a) Every matrix is row equivalent to a unique matrix in echelon form.

TRUE FALSE

(b) It is possible for the equation $A\mathbf{x} = \mathbf{b}$ to have no solutions while at the same time for the equation $A\mathbf{x} = \mathbf{0}$ to have infinitely many solutions.

TRUE FALSE

(c) If the columns of a 5 by 4 matrix are linear independent then the columns span $\mathbf{R}^5$.

TRUE FALSE

(d) If $n < m$ and $T : \mathbf{R}^n \longrightarrow \mathbf{R}^m$ is a linear transformation, then T is one-to-one.

TRUE FALSE

(e) If $\mathbf{b}$ is in the span of the columns of A, then the matrix equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

TRUE FALSE

(f) If A is matrix with a pivot point in every row and B is a matrix with a pivot point in every row, then the product AB will have a pivot point in every row.

TRUE FALSE

(g) If A contains a row of all zeroes, then the equation $A\mathbf{x} = \mathbf{0}$ has no solutions.

TRUE FALSE

(h) A homogenous system of three equations with seven unknowns will always have infinitely many solutions.

TRUE FALSE