

MA 242 – Linear Algebra
Exam #1

Name:

Instructions: To receive full credit you must show all work. Explain your answers fully and clearly. You may refer to theorems/facts in the book or from class. No calculators, books or notes of any form are allowed. Good luck!

Question	Score	Out of
1		12
2		12
3		18
4		12
5		18
6		12
7		32
Total		116

1. **(12 points)**

Consider the system of equations

$$x_1 + 3x_2 - 5x_3 = 4$$

$$x_1 + 4x_2 - 8x_3 = 7$$

$$-3x_1 - 7x_2 + 9x_3 = -6.$$

Find the general solution to these equations in parametric form. What geometric shape does the solution space form?

2. (12 points)

Are the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix},$$

linearly independent?

- (b) Give an example of 4 vectors in \mathbf{R}^3 that are linearly independent or explain (algebraically) why such an example does not exist.

3. (18 points)

(a) Define what it means for $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ to be a linear transformation.

(b) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$. Determine the matrix associated to T .

- (c) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the linear transformation defined by first rotating by $\pi/2$ radians counterclockwise and then reflecting over the x -axis. Determine the matrix associated to T .

4. (12 points)

Is \mathbf{b} in the span of the columns of A where

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} ?$$

Explain your answer.

(b) How many solutions (if any) does the equation $A\mathbf{x} = \mathbf{b}$ have where

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix} ?$$

Explain your answer.

5. (18 points)

(a) Does there exist a map from \mathbf{R}^4 to \mathbf{R}^7 that is both one-to-one and onto? If yes, give an example. If no, explain why not.

(b) Does there exist a map from \mathbf{R}^3 to \mathbf{R}^2 that is onto, but not one-to-one? If yes, give an example. If no, explain why not.

- (c) Does there exist a map from \mathbf{R}^4 to \mathbf{R}^4 that is one-to-one, but not onto? If yes, give an example. If no, explain why not.

6. (12 points)

Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ -2 & 1 \end{pmatrix}.$$

Only one of the products AB and BA makes sense. Determine which one and compute that product.

7. (32 points) Circle either TRUE or FALSE. No justification is needed.
Correct answers score 4 points each. Incorrect answers score 0 points. Leaving a question blank scores 2 points.

(a) Every matrix is row equivalent to a unique matrix in echelon form.

TRUE FALSE

(b) It is possible for the equation $A\mathbf{x} = \mathbf{b}$ to have no solutions while at the same time for the equation $A\mathbf{x} = \mathbf{0}$ to have infinitely many solutions.

TRUE FALSE

(c) If the columns of a 5 by 4 matrix are linear independent then the columns span \mathbf{R}^5 .

TRUE FALSE

(d) If $n < m$ and $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is a linear transformation, then T is one-to-one.

TRUE FALSE

(e) If \mathbf{b} is in the span of the columns of A , then the matrix equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

TRUE FALSE

(f) If A is matrix with a pivot point in every row and B is a matrix with a pivot point in every row, then the product AB will have a pivot point in every row.

TRUE FALSE

(g) If A contains a row of all zeroes, then the equation $A\mathbf{x} = \mathbf{0}$ has no solutions.

TRUE FALSE

(h) A homogenous system of three equations with seven unknowns will always have infinitely many solutions.

TRUE FALSE