MA 412 – Complex Variables
Exam #1

Name:

Instructions: To receive full credit you must show all work. Explain your answers fully and clearly. You may refer to theorems/facts in the book or from class. No calculators, books or notes of any form are allowed. Good luck!

Question	Score	Out of
1		12
2		12
3		10
4		16
5		10
6		15
7		13
8		12
Total		100

1. **(12 points)**

- Write

$$\frac{4}{-i-\sqrt{3}}$$

in exponential form

- Write

$$(-3+3i)^{27}$$

in rectangular form

2. **(12 points)**
Find the sixth roots of -4, i.e.

$$(-4)^{1/6}$$

in both exponential and rectangular form, AND sketch them

3. **(10 points)**
Sketch the region described by the inequality

$$|2\overline{z} + 6 - 6i| \geq 1$$

4. **(16 points)**

- Let D be the first quadrant, i.e. $D = \{z = x + iy | x \geq 0, y \geq 0\}$. Describe algebraically and sketch the image of D under the map $f(z) = \overline{z}^2$

- Find a region R such that the image of R under the map $f(z) = z^2$ is the set of points in the second quadrant lying between the circles $|z| = 1$ and $|z| = 4$ Give an algebraic description of R and sketch R.

5. **(10 points)**
Show that if $|z| = 3$ then

$$\left|\frac{2\overline{z}^2 - z + 4i}{z+1}\right| \leq \frac{33}{2}$$

6. **(15 points)**
Evaluate each of the following limits, or state why it does not exist.

(a)

$$\lim_{z\to\infty} \frac{3z^2 + (1+2i)z - i}{iz^2 + 4}$$

(b)

$$\lim_{z\to\infty} (2z^2 + z - 3)$$

(c)

$$\lim_{z \to \infty} \frac{z}{\overline{z}}$$

7. **(13 points)** Let

$$f(z) = (\frac{x^3}{3} + 2y) + i(\frac{y^2}{2} - 2x)$$

- Determine the set of points where the function $f(z)$ is differentiable, and calculate its derivative $f'(z)$ there.

- Determine the set of points at which $f(z)$ is analytic. Explain your reasoning.

8. **(12 points)** Give an example of a function $f(z)$ satisfying the following properties

   (a) $f(z)$ is analytic everywhere except for the three pointst $z = 2i, -2i, 1$, where it has singularities.

   (b) $f(0) = 0$, and $f(z)$ has no other zeros.

   (c) $\lim_{z\to\infty} f(z) = 4$.

   Explicitly verify that your $f(z)$ has the required properties.